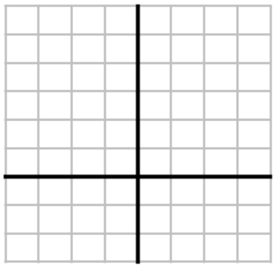
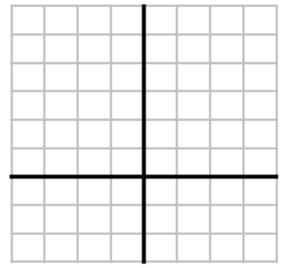
THE PARENT FUNCTIONS

LINEAR



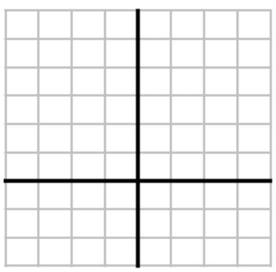
$y = x$

ABSOLUTE VALUE



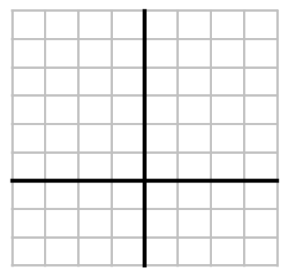
$y = |x|$

EXPONENTIAL



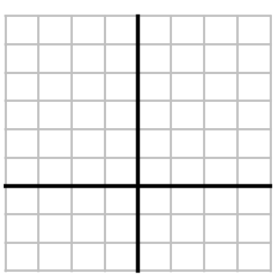
$y = 2^x$, $b > 0$

CUBE ROOT



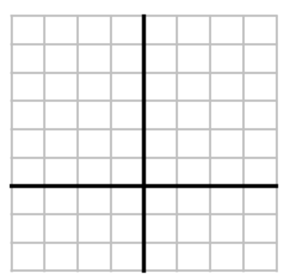
$y = \sqrt[3]{x}$

QUADRATIC



$y = x^2$

SQUARE ROOT

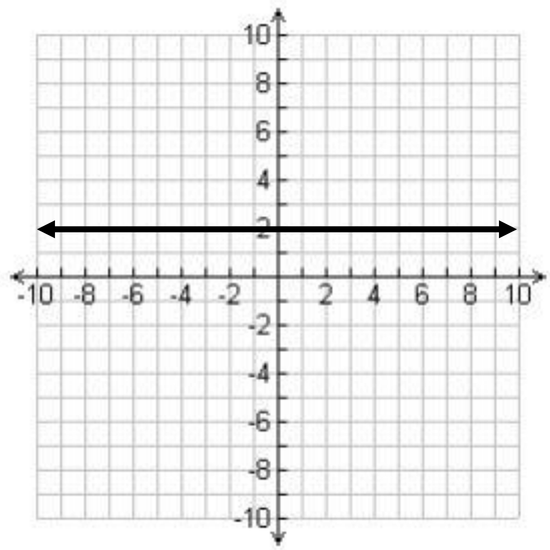


$y = \sqrt{x}$

For the graphs given below answer the questions given.

1. a. What type of function is given on the right?

   b. What is the equation of the function?

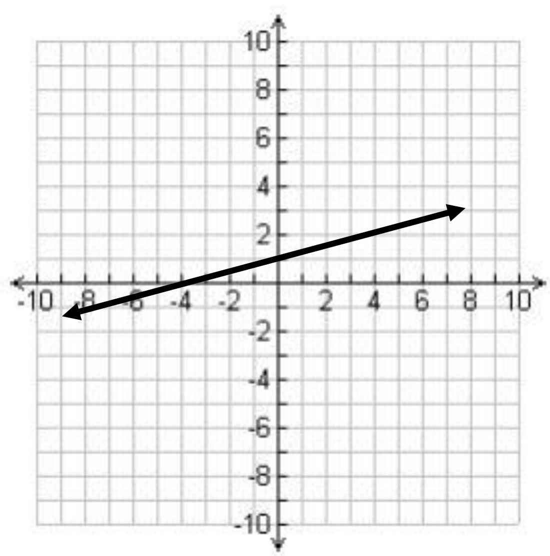


2. a. What type of function is given on the right?

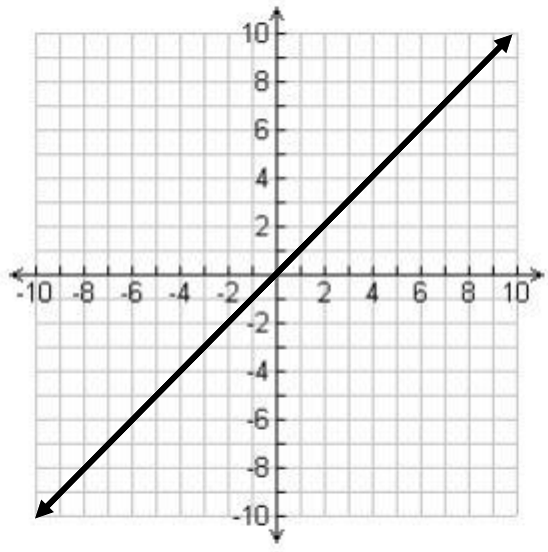
   b. What is the equation of the function?

   c. What is the x-intercept?

   d. What is the y-intercept?

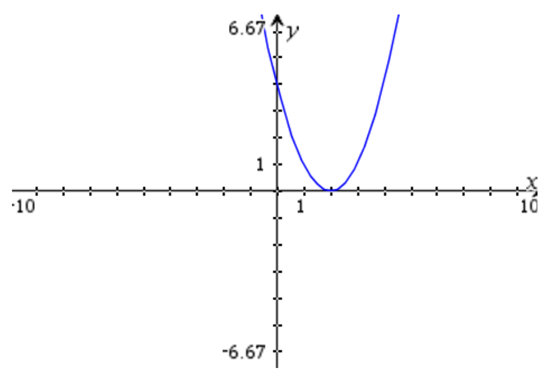


3. a. What type of function is given on the right?



b. What is the equation of the function?

4. a. Describe what happened to the parent function for the graph at the right.



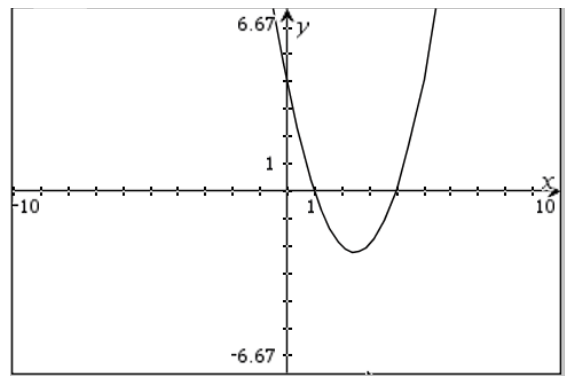
b. What is the equation of the function?

c. Write the equation in standard form.

d. What is the importance of the x-intercept in graph?

e. How many zeros of the function are there in this graph?

5. a. Describe what happened to the parent function for the graph at the right.



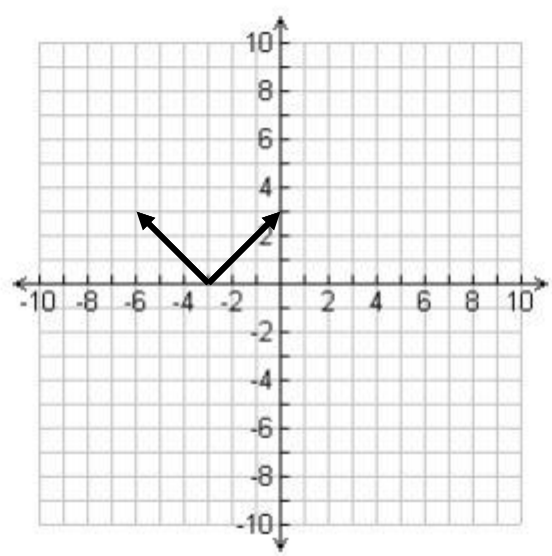
b. What is the equation of the function?

c. Write the equation in standard form.

d. What is the importance of the x-intercept in graph?

e. How many zeros of the function are there in this graph?

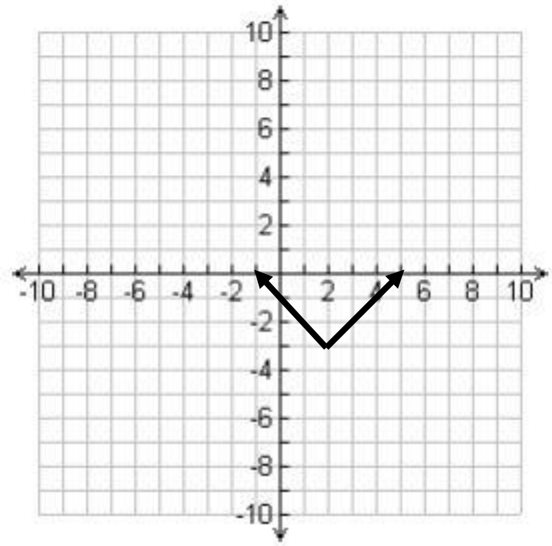
6. a. Describe what happened to the parent function for the graph at the right.



b. What is the equation of the function?

c. Write the equation in standard form.

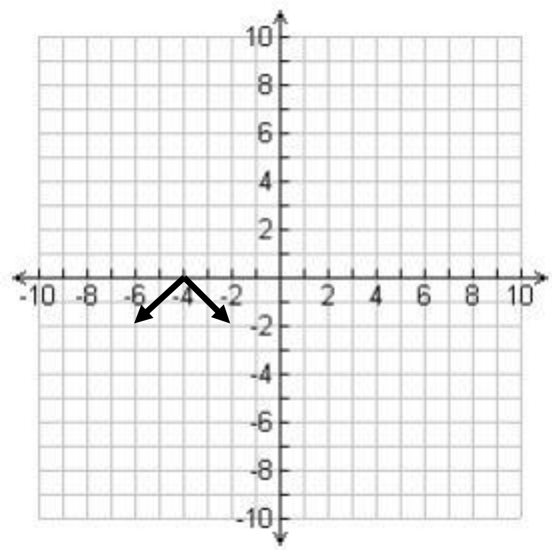
7. a. Describe what happened to the parent function for the graph at the right.



b. What is the equation of the function?

c. Write the equation in standard form.

8. a. Describe what happened to the parent function for the graph at the right.



b. What is the equation of the function?

c. Write the equation in standard form.

Parent Function Worksheet

 # 1- 7 Give the name of the parent function and describe the transformation represented.

1. $g(x) = x^2 - 1$ Name: ____

   Transformation: __________________________________

2. $f(x) = 2|x-1|$ Name: ____

   Transformation: __________________________________

3. $h(x) = \sqrt{x-2}$ Name: ____

   Transformation: __________________________________

4. $g(x) = x^2 + 3$ Name: ____

   Transformation: __________________________________

5. $g(x) = -3^x$ Name: ____

   Transformation: __________________________________

6. $f(x) = |x+5| - 2$ Name: ____

   Transformation: __________________________________

7. $h(x) = x+6$ Name: ____

   Transformation: __________________________________

#8-12 Identify the domain and range of the function. Describe the transformation from its parent function.

8. $g(x) = 3\sqrt{x}$ Domain : ______________________ Range : ____________________

   Transformation: ____

9. $h(x) = -x^2 + 1$ Domain : ______________________ Range : ____________________

   Transformation: ____

10. $h(x) = -|x-2|$ Domain : ______________________ Range : ____________________

Transformation: ____

11. $f(x) = \frac{3}{4}\sqrt{x}$ Domain : ______________________ Range : ____________________

Transformation: ____

12. $h(x) = 6(x + 9)^2$ Domain : ______________________ Range : ____________________

Transformation: ____

#13 - 17 Given the parent function and a description of the transformation, write the equation of the transformed function, f(x).

13. Absolute value—vertical shift up 5, horizontal shift right 3. _____________________

14. Radical—vertical compression by $\frac{2}{5}$ _____________________

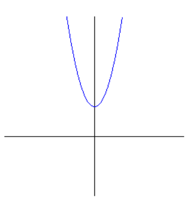
15. Quadratic—reflected over the x axis and vertical shift down 2 _____________________

16. Linear—vertical stretch by 8 _____________________

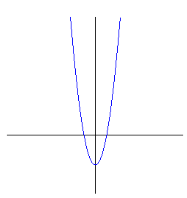
17. Quadratic—vertical compression by .45, horizontal shift left 8. _____________________

18. Which graph best represents the function $f(x) = 2x^2 - 2$? _________________

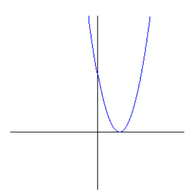
a.



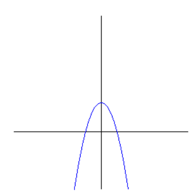
b.



c.



d.



What type of relationship is indicated by the given set of ordered pairs?
How do you know? Explain or show your reasoning.

X	Y
0	-2
1	1
2	4
3	7
4	10
5	13

x	$ax+b$	First Difference
0		
1		
2		
3		
4		
5		

What type of relationship is indicated by the given set of ordered pairs?

X	Y	First Difference	Second Difference
0	-2		
1	0		
2	4		
3	10		
4	18		
5	28		

x	$ax^2 + bx + c$	First Difference	Second Difference
0			
1			
2			
3			
4			
5			

1. Create a table to find the second differences for the polynomial $16 - 4t^2$ for integer values of t from 0 to 5.

x	$16-4t^2$	First Difference	Second Difference
0			
1			
2			
3			
4			
5			

2. Show that the set of ordered pairs (x, y) in the table below satisfies a linear relationship. Find the equation of the form y = ax + b that all of the ordered pairs satisfy.

X	Y	First Difference
0	4	
1	1	
2	-2	
3	-5	
4	-8	
5	-11	

3. Show that the set of ordered pairs (x, y) in the table below satisfies a quadratic relationship. Find the equation of the form $y = ax^2 + bx + c$ that all of the ordered pairs satisfy.

X	Y	First Difference	Second Difference
0	1		
1	2		
2	-1		
3	-8		
4	-19		
5	-34		

4. (a) What type of relationship is indicated by the following set of ordered pairs?

X	Y
0	0
1	1
2	6
3	15
4	28
5	45

(b) Find the equation that all ordered pairs above satisfy.

Module 1 – Polynomial, Rational, and Radical Relationships

For each problem answer the following questions:

a. What type of relationship is indicated by the following set of ordered pairs?
b. Find an equation that all ordered pairs satisfies.

1.

X	Y
0	36
1	20
2	-28
3	-108
4	-220
5	-364

2.

X	Y
0	5
1	4
2	-1
3	-10
4	-23
5	-40

3.

X	Y
0	0
1	2
2	10
3	24
4	44

MATHCING POLYNOMIALS

Name______________________________

Part 1 – Sum and Difference

1. $(4 + 2x) - (3x - x^2 + 6)$

2. $(-x + 1) + (2x - 3)$

3. $(6x^2 - 3x - 1) - (x^2 - 5x - 6)$

4. $(4x - 2) - (2x - 1)$

5. $(2x^3 + x + 4) - (2x^3 - x^2 + 2x - 1)$

6. $(3x + 2) - (2x - 4)$

7. $(3x^2 + 2x + 5) + (2x^2 + 2)$

8. $x + (x - 3)$

9. $(x^3 + x^2 + x + 1) + (-x^3 - 2x^2 - 4x - 4)$

10. $(x^2 + 2x) + (3 - 3x)$

a. $2x – 1$

b. $x^2 - x + 3$

c. $x + 6$

d. $5x^2 + 2x + 7$

e. $x^2 - x – 2$

f. $x – 2$

g. $-x^2 - 3x – 3$

h. $5x^2 + 2x + 5$

i. $2x – 3$

j. $x^2 - x + 5$

Part 2 - Product

11. $8x^2 - 8x + 2$

12. $(x - 2)(x - 3)$

13. $(x + 3)^2$

14. $x^2 + 5x + 6$

15. $2x(2x^2 - 2x - 1)$

16. $(2x + 1)(2x + 1)$

17. $4x^2 – 1$

18. $(4x + 1)(2x - 1)$

19. $x(4x^2 + 4x + 1)$

20. $(x^2 + 2)(x^2 - 2)$

k. $(x + 3)(x + 2)$

l. $4x^2 + 4x + 1$

m. $4x^3 + 4x^2 + x$

n. $(4x - 2)(2x - 1)$

o. $x^2 - 5x + 6$

p. $8x^2 - 2x – 1$

q. $(2x + 1)(2x - 1)$

r. $x^2 + 6x + 9$

s. $4x^3 - 4x^2 - 2x$

t. $x^4 – 4$

Multiply and combine like terms to write as the sum or difference of monomials.

1. $(g + 5) + (2g + 7)$

2. $(5d + 5) - (d + 1)$

3. $(x^2 - 3x - 3) + (2x^2 + 7x - 2)$

4. $(-2f^2 - 3f - 5) + (-2f^2 - 3f + 8)$

5. $-5(2c^2 - d^2)$

6. $x^2(2x + 9)$

7. $(a - 5)^2$

8. $(2x - 3)(3x - 5)$

9. $(r - 2t)(r + 2t)$

10. $(3y + 4)(2y - 3)$

11. $(3 - 2b)(3 + 2b)$

12. $(3w + 1)^2$

13. $(3n^2 + 1) + (8n^2 - 8)$

14. $(6w - 11w^2) - (4 + 7w^2)$

15. $(w + 2t)(w^2 - 2wt + 4t^2)$

16. $(x + y)(x^2 - 3xy + 2y^2)$

17. $2a(5 + 4a)$

18. $x^3(x + 6) + 9$

19. $\frac{1}{8}(96z + 24z^2)$

20. $2^{23}(2^{84} - 2^{81})$

21. $(x - 4)(x + 5)$

22. $(10w - 1)(10w + 1)$

23. $(3z^2 - 8)(3z^2 + 8)$

24. $(-5w - 3)w^2$

25. $(t - 1)(t + 1)(t^2 + 1)$

26. $(2r + 1)(2r^2 + 1)$

27. $(x+2)(x+2)(x+2)$

28. $(w - 1)(w^5 + w^4 + w^3 + w^2 + w + 1)$

29. $n(n + 1)(n + 2)(n + 3)$

30. $n(n + 1)(n + 2)$

31. $(x + 1)(x^3 - x^2 + x - 1)$

32. $n(n + 1)(n + 2)(n + 3)(n + 4)$

33. $(m^3 - 2m + 1)(m^2 - m + 2)$

34. $(x + 1)(x^5 - x^4 + x^3 - x^2 + x - 1)$

35. $(x + 1)(x^7 - x^6 + x^5 - x^4 + x^3 - x^2 + x - 1)$

Factor each polynomial completely. If it can't be factored write "Not Factorable".

1. $m^2 + 7m - 18$

2. $2x^2 - 3x - 5$

3. $4z^2 + 4z - 15$

4. $4p^2 + 4p - 24$

5. $3y^2 + 21y + 36$

6. $c^2 - 100$

7. $r^3 + 3r^2 - 54r$

8. $8a^2 + 2a - 6$

9. $c^2 - 49$

10. $16r^2 - 169$

11. $b^4 - 81$

12. $25x^2 - 64y^2$

13. $3x^2 + 12x$

14. $x^2 - 12x + 27$

15. $2x^2 - x - 6$

16. $-x^2 - 15x - 56$

17. $4a^2 + 9a + 2$

18. $9x^2 - 12x + 4$

20. $3a^2 - 27a + 60$

21. $36x^2 - 64y^2$

22. $75 - 3x^2$

23. $28x^2 + 12x + 42x + 18$

24. $2 + 5x + 2x^2$

25. $4x^2 + 9$

26. $5x^2 - 20$

27. $5a^2 - a - 18$

28. $6x^2 + 7x$

29. $(3x)^2 - 5^2$

Factor by Grouping: Factor each expression completely.

1. $8r^3 - 64r^2 + r - 8$

2. $12p^3 - 21p^2 + 28p - 49$

3. $12x^3 + 2x^2 - 30x - 5$

4. $6v^3 - 16v^2 + 21v - 56$

5. $63n^3 + 54n^2 - 105n - 90$

6. $21k^3 - 84k^2 + 15k - 60$

7. $25v^3 + 5v^2 + 30v + 6$

8. $105n^3 + 175n^2 - 75n - 125$

9. $96n^3 - 84n^2 + 112n - 98$

10. $28v^3 + 16v^2 - 21v - 12$

11. $4v^3 - 12v^2 - 5v + 15$

12. $49x^3 - 35x^2 + 56x - 40$

13. $24p^3 + 15p^2 - 56p - 35$

14. $24r^3 - 64r^2 - 21r + 56$

15. $56xw + 49xk^2 - 24yw - 21yk^2$

16. $42mc + 36md - 7n^2c - 6n^2d$

17. $12x^2u + 3x^2v + 28yu + 7yv$

18. $40ac^2 + 25ak^2 + 32bc^2 + 20bk^2$

19. $12bc - 4bd - 15xc + 5xd$

20. $16mn - 4m^2 + 28n - 7m$

21. $56xy - 35x + 16ry - 10r$

22. $21xy + 15x + 35ry + 25r$

23. $5a^2z - 4a^2c + 15xz - 12xc$

24. $4xy + 6 - x - 24y$

25. $21xy - 12b^2 + 14xb - 18by$

26. $9mz - 4nc + 3mc - 12nz$

27. $28xy + 25 + 35x + 20y$

28. $30uv + 30u + 36u^2 + 25v$

Factor each of the following perfect square trinomials. In the last two problems, look for a greatest common factor to remove first.

1) $a^2 + 4a + 4$

2) $p^2 + 2p + 1$

3) $x^2 - 10x + 25$

4) $y^2 - 8y + 16$

5) $r^2 + 24r + 144$

6) $k^2 + 121 + 22k$

7) $y^2 - 20y + 100$

8) $49 + 14a + a^2$

9) $9x^2 + 24x + 16$

10) $16t^2 - 40t + 25$

11) $25k^2 - 20k + 4$

12) $4c^2 + 12c + 9$

13) $100 - 20m + m^2$

14) $64y^2 - 48ya + 9a^2$

15) $100a^2 - 140ab + 49b^2$

16) $49x^2 + 28xy + 4y^2$

17) $x^3y + 6x^2y^2 + 9xy^3$

18) $4k^3w + 20k^2w^2 + 25kw^3$

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $a + 8$

2. $(2x - 1)(4x^2 + 3)$

3. $-5x^5 + 3x^3 - 8$

4. $18 - 3y + 5y^2 - y^5 + 7y^6$

5. $u^3 + 4u^2t^2 + t^4$

6. $2r - r^2 + \frac{1}{r^2}$

Find *p*(–1) and *p*(2) for each function.

7. $p(x) = 4 - 3x$

8. $p(x) = 3x + x^2$

9. $p(x) = 2x^2 - 4x + 1$

10. $p(x) = -2x^3 + 5x + 3$

11. $p(x) = x^4 + 8x^2 - 10$

12. $p(x) = \frac{1}{3}x^2 - \frac{2}{3}x + 2$

If $p(x) = 4x^2 - 3$ and $r(x) = 1 + 3x$, find each value.

13. $p(a)$

14. $r(2a)$

15. $3r(a)$

16. $-4p(a)$

17. $p(a^2)$

18. $r(x + 2)$

Module 1 – Polynomial, Rational, and Radical Relationships

State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.

1. $(3x^2 + 1)(2x^2 - 9)$

2. $\frac{1}{5}a^3 - \frac{3}{5}a^2 + \frac{4}{5}a$

3. $\frac{2}{m^2} + 3m - 12$

4. $27 + 3xy^3 - 12x^2y^2 - 10y$

Find *p*(–2) and *p*(3) for each function.

5. $p(x) = x^3 - x^5$

6. $p(x) = -7x^2 + 5x + 9$

7. $p(x) = -x^5 + 4x^3$

8. $p(x) = 3x^3 - x^2 + 2x - 5$

9. $p(x) = x^4 + \frac{1}{2}x^3 - \frac{1}{2}x$

10. $p(x) = \frac{1}{3x^3} + \frac{2}{3x^2} + 3x$

If $p(x) = 3x^2 - 4$ and $r(x) = 2x^2 - 5x + 1$, find each value.

11. $p(8a)$

12. $r(a^2)$

13. $-5r(2a)$

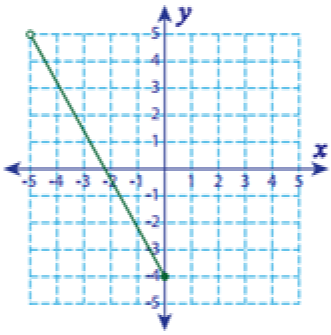
14. $r(x + 2)$

15. $p(x^2 - 1)$

16. $5p(x + 2)$

Find the domain and range for each graph.

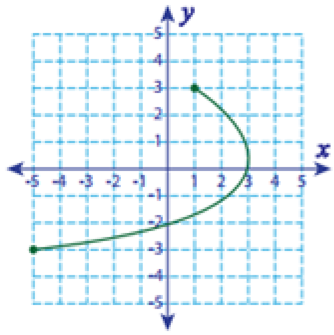
1)



Domain : ____________

Range : ____________

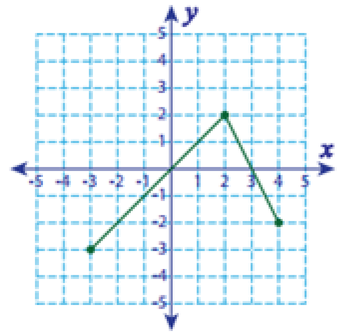
2)



Domain : ____________

Range : ____________

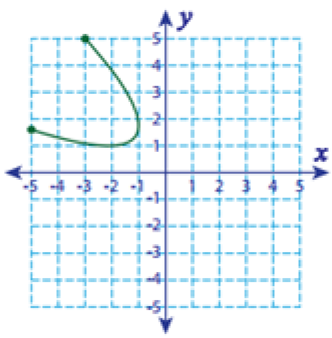
3)



Domain : ____________

Range : ____________

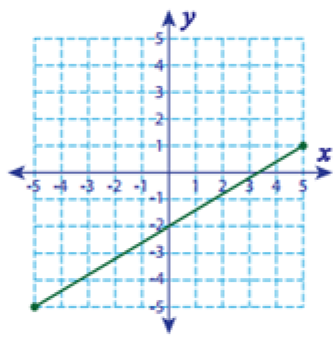
4)



Domain : ____________

Range : ____________

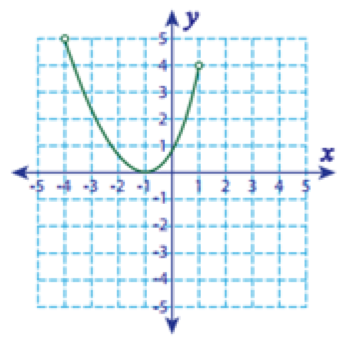
5)



Domain : ____________

Range : ____________

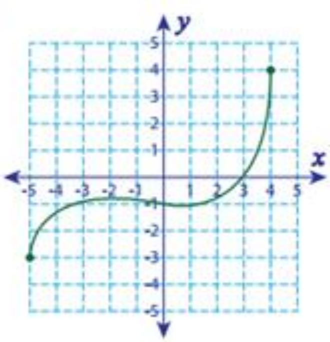
6)



Domain : ____________

Range : ____________

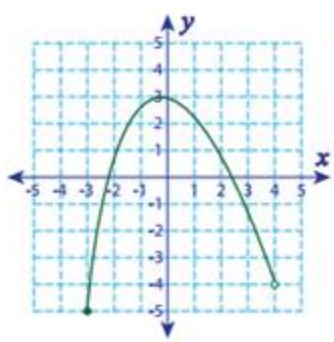
7)



Domain : ____________

Range : ____________

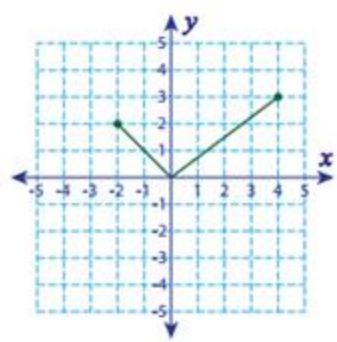
8)



Domain : ____________

Range : ____________

9)

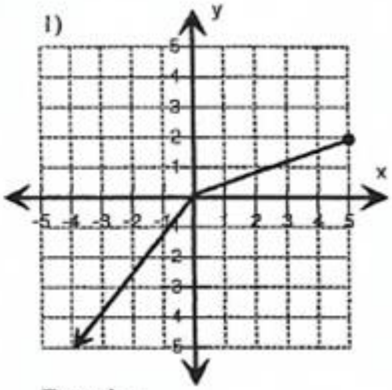


Domain : ____________

Range : ____________

Find the Domain and Range for each graph.

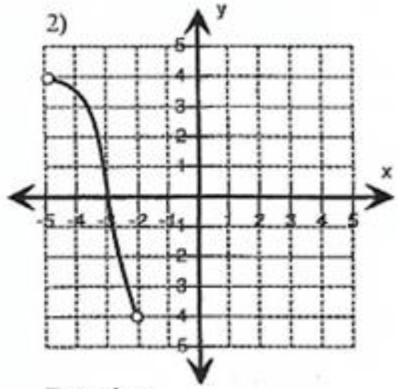
1)



Domain : ______________

Range : ______________

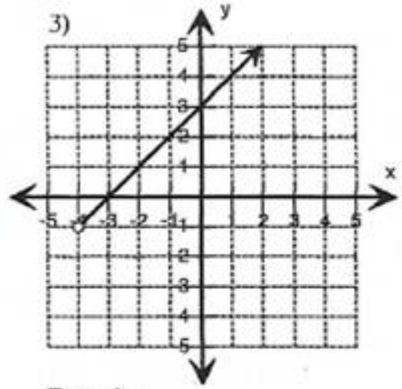
2)



Domain : ______________

Range : ______________

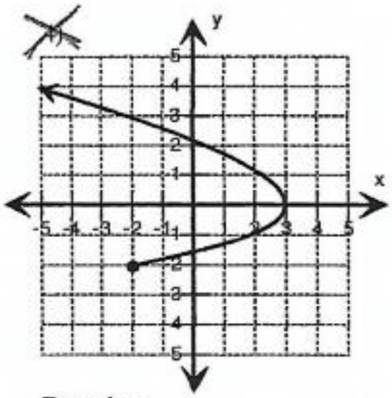
3)



Domain : ______________

Range : ______________

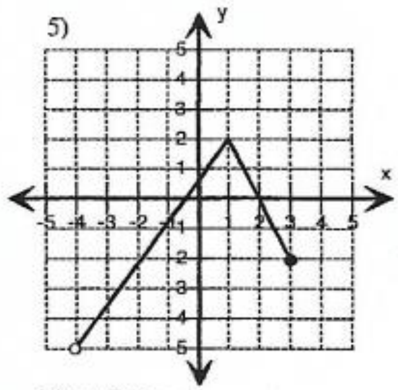
4)



Domain : ______________

Range : ______________

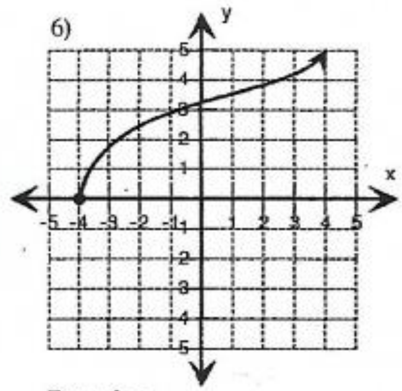
5)



Domain : ______________

Range : ______________

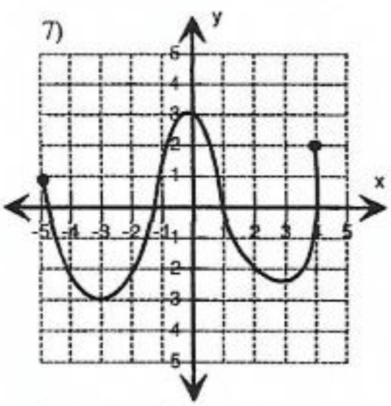
6)



Domain : ______________

Range : ______________

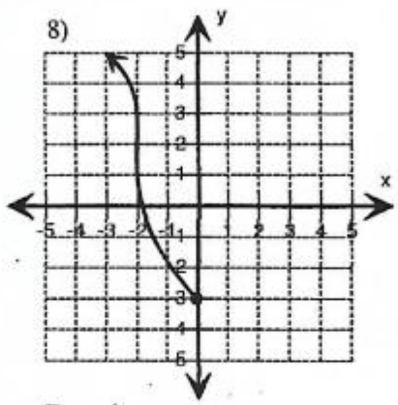
7)



Domain : ______________

Range : ______________

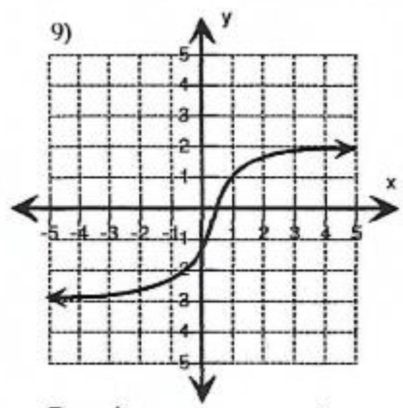
8)



Domain : ______________

Range : ______________

9)

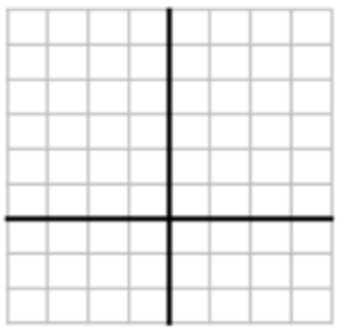


Domain : ______________

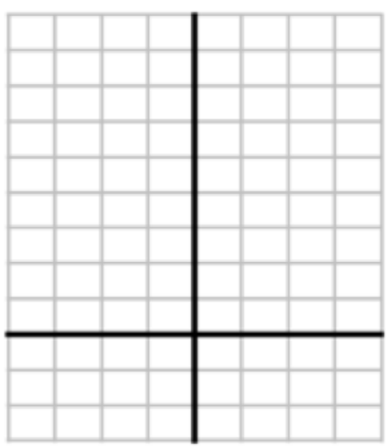
Range : ______________

WHAT CAN GRAPHS TELL US?

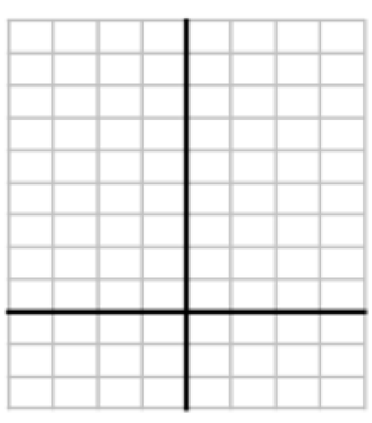
LINEAR



EXPONENTIAL



QUADRATIC



Module 1 – Polynomial, Rational, and Radical Relationships

The different forms of linear and quadratic functions are listed below. Explain how the structure of each form gives you information about the graph of the function.

LINEAR

Standard form: $ax + by = c$

Slope-intercept form: $y = mx + b$

Point-slope form: $y = m(x - x_1) + y_1$

QUADRATIC

Standard form: $y = ax^2 + bx + c$

Factored form: $y = a(x - r_1)(x - r_2)$

Vertex form: $y = a(x - h)^2 + k$

For each, write what you know about the function and then graph

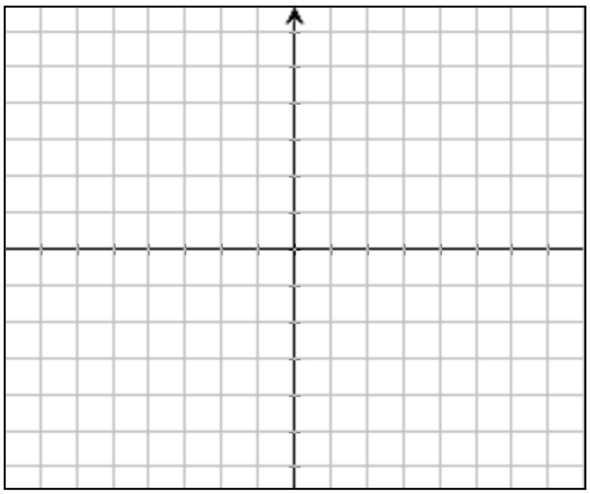
1. $f(x) = (x-2)(x+4)$

What I know about this function:

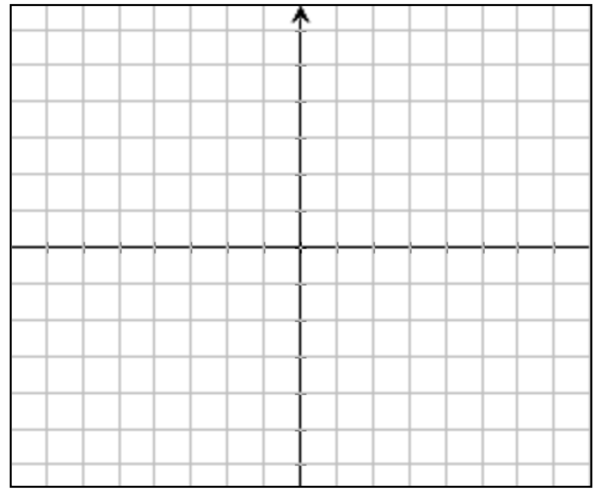
End Behavior:

as $x \to -\infty$, $f(x) \to$ ______

as $x \to \infty$, $f(x) \to$ ______



2. $h(x) = 2(x - 3) + 1$



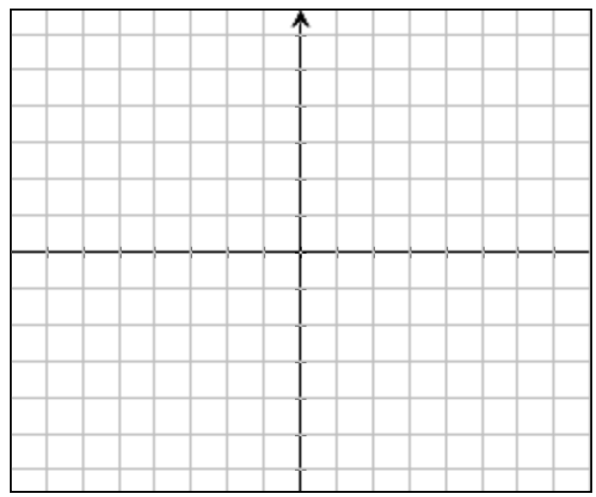
What I know about this function:

End Behavior:

as $x \to -\infty$, $f(x) \to$ ______

as $x \to \infty$, $f(x) \to$ ______

3. $g(x) = x^2 + 4x - 5$



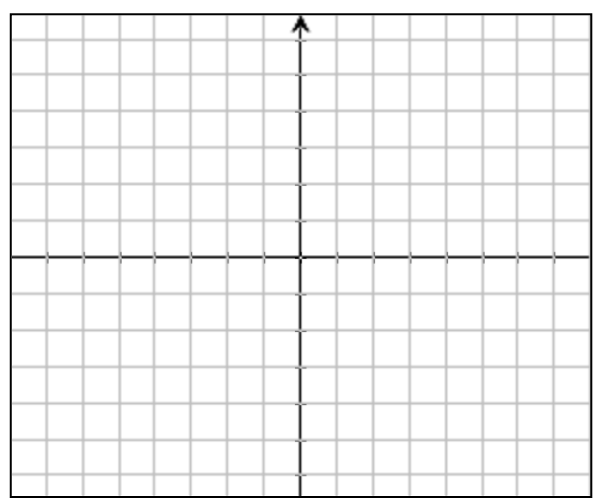
What I know about this function:

End Behavior:

as $x \to -\infty$, $f(x) \to$ ______

as $x \to \infty$, $f(x) \to$ ______

4. $g(x) = -2(x + 3)^2 + 3$



What I know about this function:

End Behavior:

as $x \to -\infty$, $f(x) \to$ ______

as $x \to \infty$, $f(x) \to$ ______

Arrange the **EXPRESSIONS** in order from the **least** to the **greatest** when the value of x is zero.

2^x

$x^2 - 20$

$x^5 - 4x^2 + 1$

$x + 30$

$x^4 - 1$

$x^3 + x^2 - 4$

$-x^2 + 3x$

Do you think this order would change when x represents other numbers?

Order each expression from **LEAST** to **GREATEST** when x represents a very large number. (so large, it is "close to" or approaching positive infinity) Evaluate each for x = 10.

2^x

$x^2 - 20$

$x^5 - 4x^2 + 1$

$x + 30$

$x^4 - 1$

$x^3 + x^2 - 4$

$-x^2 + 3x$

Order each expression from **GREATEST** to **LEAST** when x represents a number that is approaching negative infinity. Evaluate each for x = -10.

2^x

$x^2 - 20$

$x^5 - 4x^2 + 1$

$x + 30$

$x^4 - 1$

$x^3 + x^2 - 4$

$-x^2 + 3x$

What do you notice about the end behavior of these functions?

x^2

x^3

x^4

x^5

x^6

x^7

What would happen if I reflected them over the x-axis?

$-x^2$

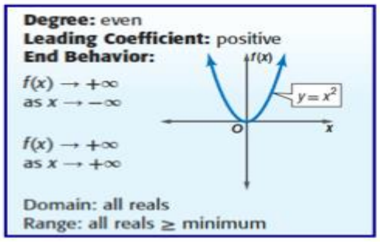
$-x^3$

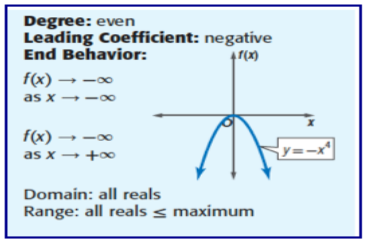
$-x^4$

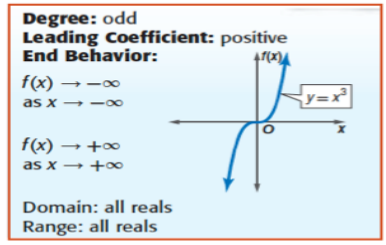
$-x^5$

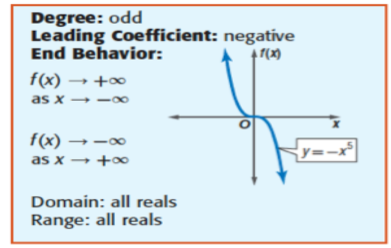
$-x^6$

$-x^7$

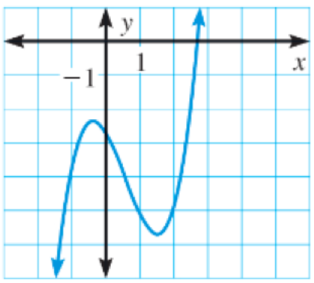


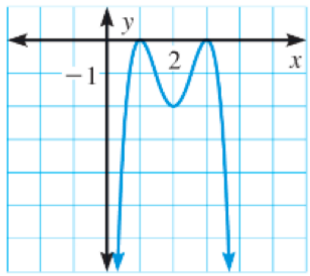


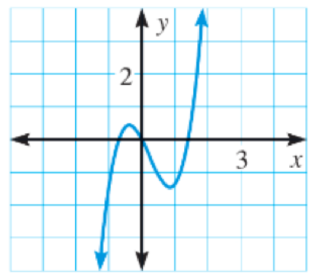


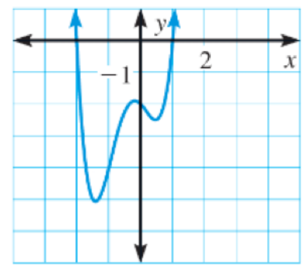


#1 #2 #3 #4









Fill in the following information for each graph given above.

#1 Degree

Zeros

End Behavior

#2 Degree

Zeros

End Behavior

#3 Degree

Zeros

End Behavior

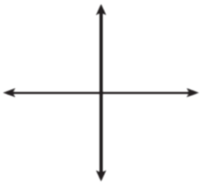
#4 Degree

Zeros

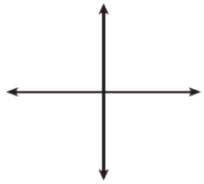
End Behavior

Use your calculator and make a sketch of each function on the axes below.

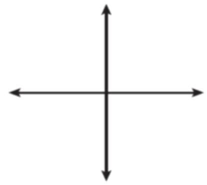
$f(x)=x^2$



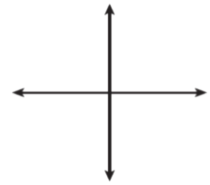
$f(x)=x^3$



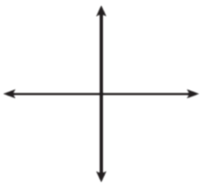
$f(x)=x^4$



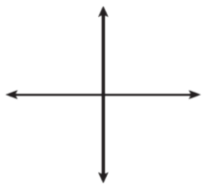
$f(x)=x^5$



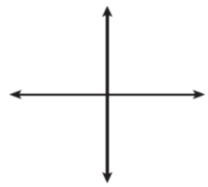
$f(x)=-x^2$



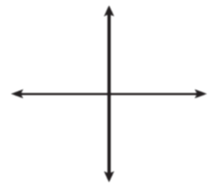
$f(x)=-x^3$



$f(x)=-x^4$



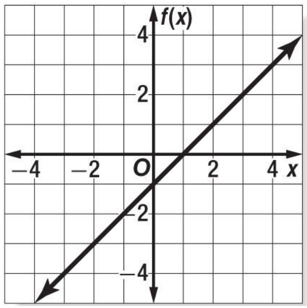
$f(x)=-x^5$



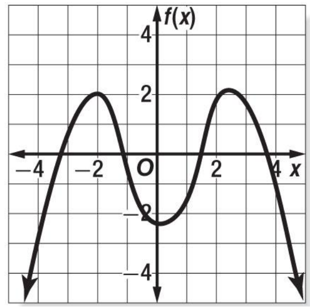
	WORDS	**PICTURE**	**NOTATION**
EVEN DEGREE $a>0$			
EVEN DEGREE $a<0$			
ODD DEGREE $a>0$			
ODD DEGREE $a<0$			

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd-degree or an even-degree function, and
c. state the number of real zeroes.

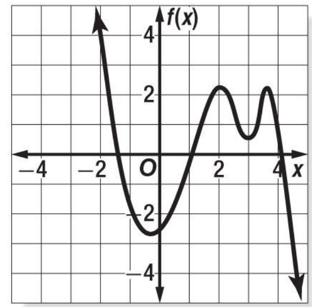
1.



2.



3.



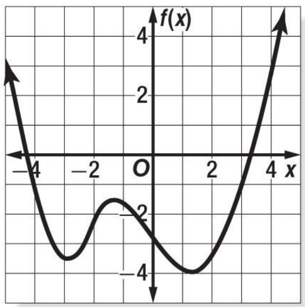
1.

2.

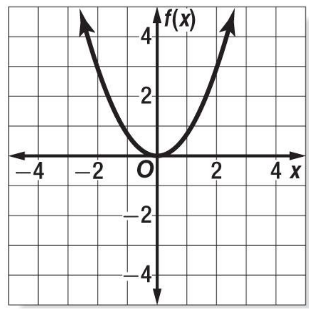
3.

For each graph,
a. describe the end behavior,
b. determine whether it represents an odd–degree or an even-degree function, and
c. state the number of real zeroes.

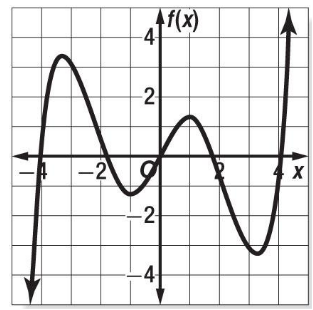
4.



5.



6.



4.

5.

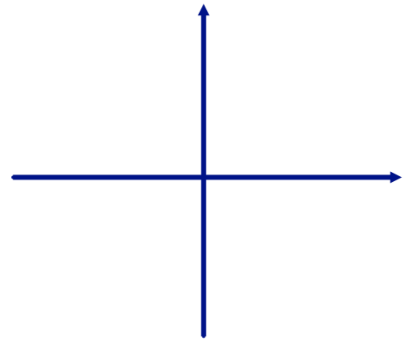
6.

Module 1 – Polynomial, Rational, and Radical Relationships

1. Complete the table and draw a sketch for each of the following functions:

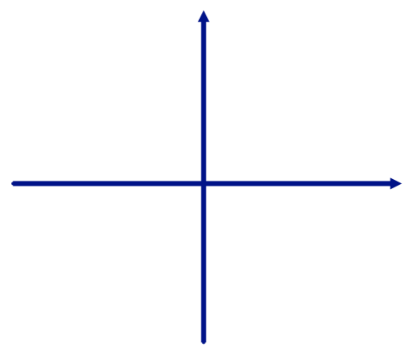
(a) $f(x)=\frac{1}{2}(x+5)(x+1)(x-2)$

Leading Term *a* = degree?	End Behavior		
Zeros			
y-intercept			



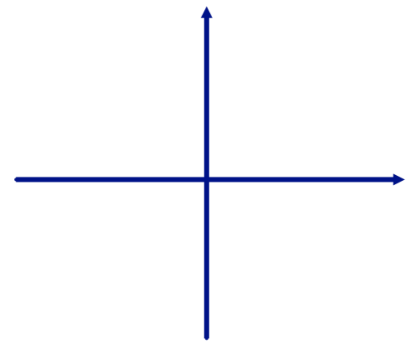
(b) $g(x)=-(x+2)(x-4)(x-6)$

Leading Term *a* = degree?	End Behavior		
Zeros			
y-intercept			



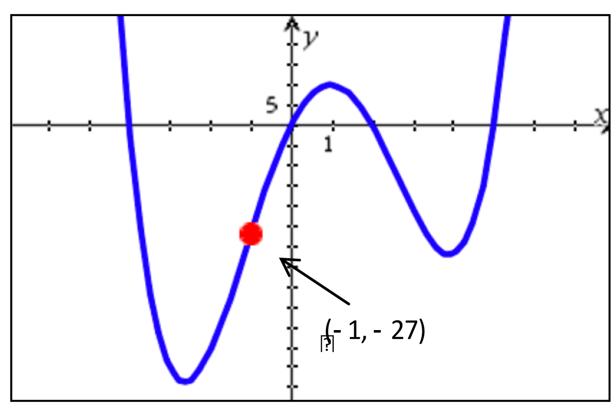
(c) $h(x) = x^3 - 2x^2 - x + 2$

Leading Term *a* = degree?	End Behavior		
Zeros			
y-intercept			



2. Use the graph on the right to complete the table.

Leading Term *a* = degree?	End Behavior
Zeros	
y-intercept	
Function	



Module 1 – Polynomial, Rational, and Radical Relationships

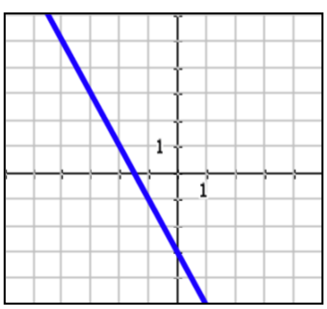
I. For each situation:

- Determine the function type. If it is a polynomial, state the degree of the polynomial and whether it is an even or an odd degree polynomial.
- For each, state the end behavior based on your knowledge of the function. Use the format: As $x \to -\infty,\ f(x) \to$ _____ and as $x \to \infty,\ f(x) \to$ _____

(1) $f(x) = 2 + 3x$	(2) $f(x) = x^4 - 81$	(3) $f(x) = 4^x$
(4) $f(x) = x^3 + 2x^2 - x + 5$	(5) $f(x) = -2x^3 + 3x^2 - x + 7$	(6) $f(x) = -5(x-2)(x+1)$
(7) $f(x) = \sqrt{x} - 2$	(8) $f(x) = 3(x-1)(x+2)(x-3)$	(9) $f(x) = -.5(x+4)^2 - 2$

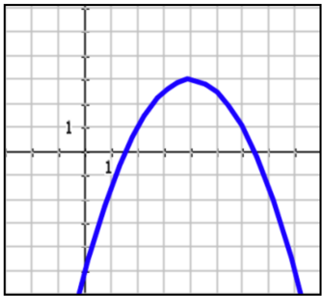
Use the graphs below to describe the end behavior of each function. Use the same format as above.

(10)

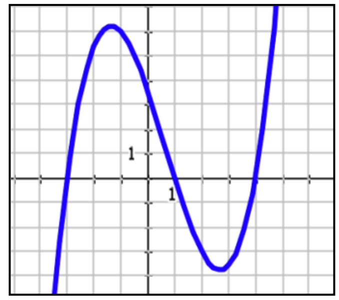


(11)

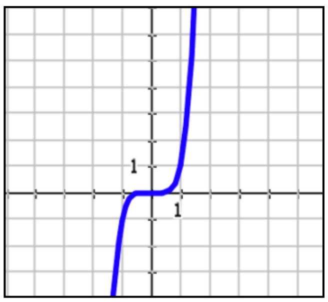
(12)



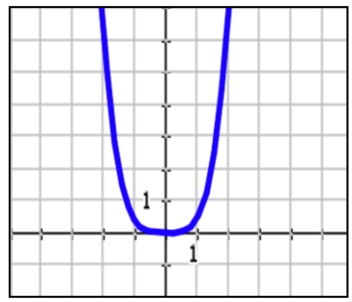
(13)



(14)



(15)

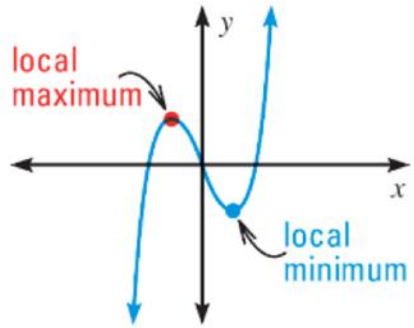


II Use the functions from problems 1 - 15 to answer the following WITHOUT finding the solution to each problem. Write a short explanation for each answer.

(16) Compare problems 4 and 5: Which has the greatest value as $x \to \infty$?	(17) Compare problems 7 and 12: Which has the greatest value as $x \to \infty$?
(18) Compare problems 2 and 8: Which of these two polynomials has the highest degree?	(19) Compare problems 7 and 13: Which has the greatest average rate of change from [10, 15]?
(20) Compare problems 11 and 14: Which grows faster as $x \to \infty$?	EXTRA: Create your own comparison problem to ask the class (be sure you know the answer).

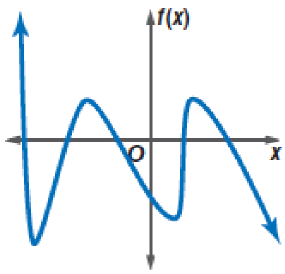
Another important characteristic of graphs of polynomial functions is that they have ***turning points*** corresponding to local maximum and minimum values.

- The *y*-coordinate of a turning point is a local maximum if the point is higher than all nearby points.
- The *y*-coordinate of a turning point is a local minimum if the point is lower than all nearby points.



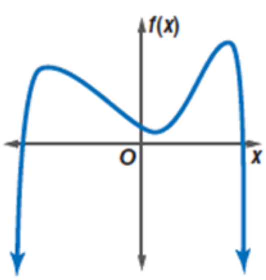
Putting It All Together: Use the graphs and equation below to answer the questions displayed on the SmartBoard.

1.



2. $f(x) = x^3 + 3x^2 - 4x$

3.



Module 1 – Polynomial, Rational, and Radical Relationships

1. $f(x) = x(x-1)(x+1)$

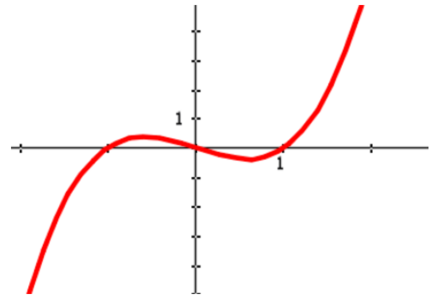
   Zeros

   Solutions to $f(x) = 0$

   x-intercepts

   Is the degree equal or less than or greater than

   - the number of x-intercepts?
   - the number of relative max/min?



2. $f(x) = (x+3)(x+3)(x+3)(x+3)$

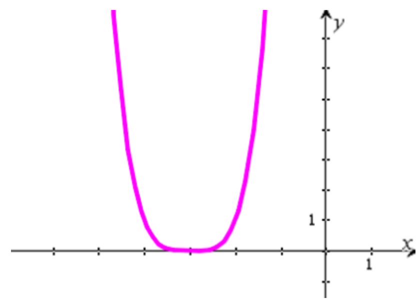
   Zeros

   Solutions to $f(x) = 0$

   x-intercepts

   Is the degree equal or less than or greater than

   - the number of x-intercepts?
   - the number of relative max/min?



3. $f(x) = (x-1)(x-2)(x+3)(x+4)(x+4)$

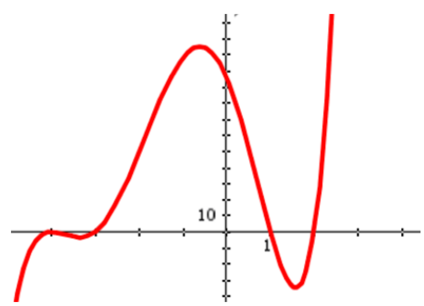
   Zeros

   Solutions to $f(x) = 0$

   x-intercepts

   Is the degree equal or less than or greater than

   - the number of x-intercepts?
   - the number of relative max/min?



4. $f(x) = (x^2+1)(x-2)(x-3)$

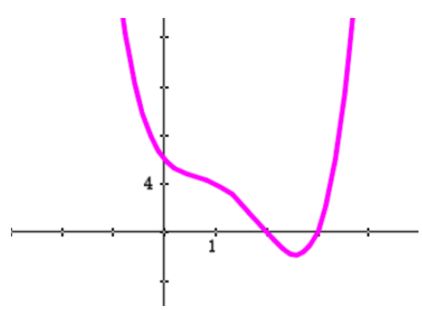
   Zeros

   Solutions to $f(x) = 0$

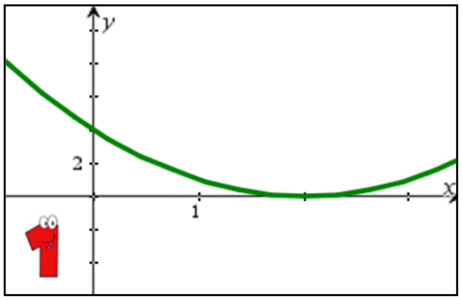
   x-intercepts

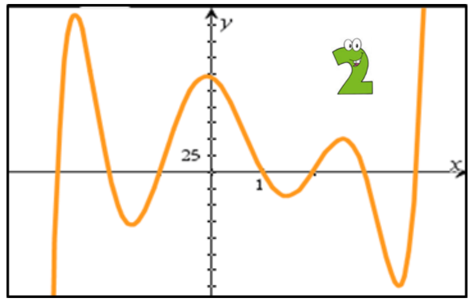
   Is the degree equal or less than or greater than

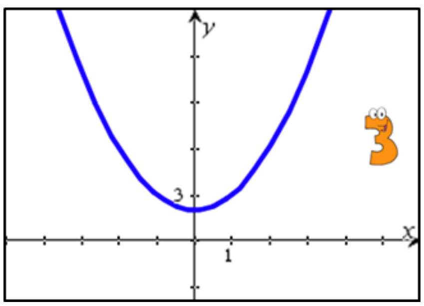
   - the number of x-intercepts?
   - the number of relative max/min?

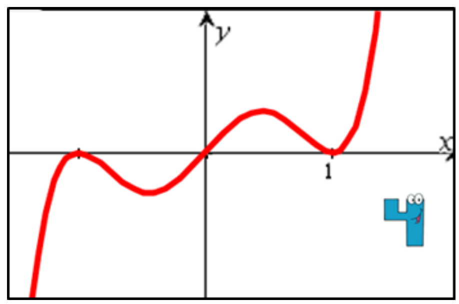


Match each function to its graph. Be ready to defend your choice!









(a) $f(x) = x(x + 1)^2(x - 1)^2$

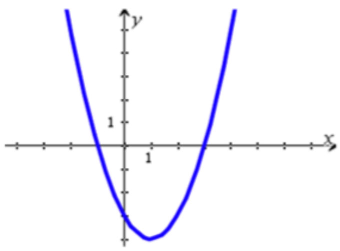
(b) $f(x) = (x - 2)^2$

(c) $f(x) = (x^2 + 2)^2$

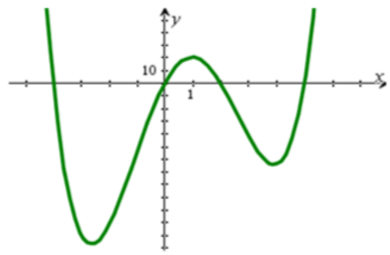
(d) $f(x) = (x - 1)(x + 1)(x - 2)(x + 2)(x - 3)(x + 3)(x - 4)$

Complete the chart for each function given below.

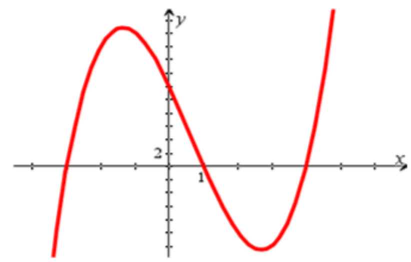
$f(x) = (x+1)(x-3)$



$h(x) = x(x+4)(x-2)(x-5)$



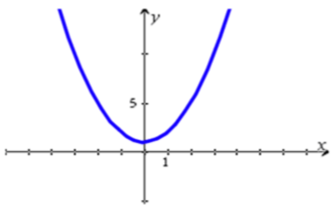
$g(x) = (x+3)(x-1)(x-4)$



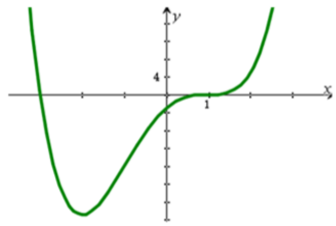
	$f(x)$	$g(x)$	$h(x)$
Degree of Polynomial			
Number of x-intercepts in each graph			
Number of relative maxima or minima in each graph			
What observations can we make from this information?			

Complete the chart for each function given below.

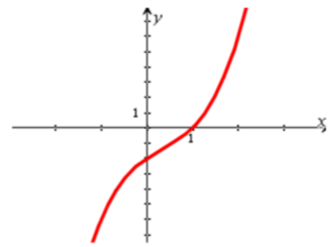
$f(x)=x^2+1$



$h(x)=(x+3)(x-1)(x-1)(x-1)$



$g(x)=(x^2+2)(x-1)$



	$f(x)$	$g(x)$	$h(x)$
Degree of Polynomial			
Number of x-intercepts in each graph			
Number of relative maxima or minima in each graph			
What observations can we make from this information?			

Use the long division algorithm to determine the quotient. Do the work in your notebook.

1. $\dfrac{2x^3-13x^2-x+3}{2x+1}$

2. $\dfrac{3x^3+4x^2+7x+22}{x+2}$

3. $\dfrac{x^4+6x^3-7x^2-24x+12}{x^2-4}$

4. $\dfrac{2x^2-3x-5}{x+1}$

5. $\dfrac{2x^4+14x^3+x^2-21x-6}{2x^2-3}$

6. $\dfrac{2x^2+x-3}{x-1}$

7. $(n^2 + 7n + 10) \div (n + 5)$

8. $(d^2 + 4d + 3) \div (d + 1)$

9. $(2t^2 + 13t + 15) \div (t + 5)$

10. $(6y^2 + y - 2)(2y - 1)^{-1}$

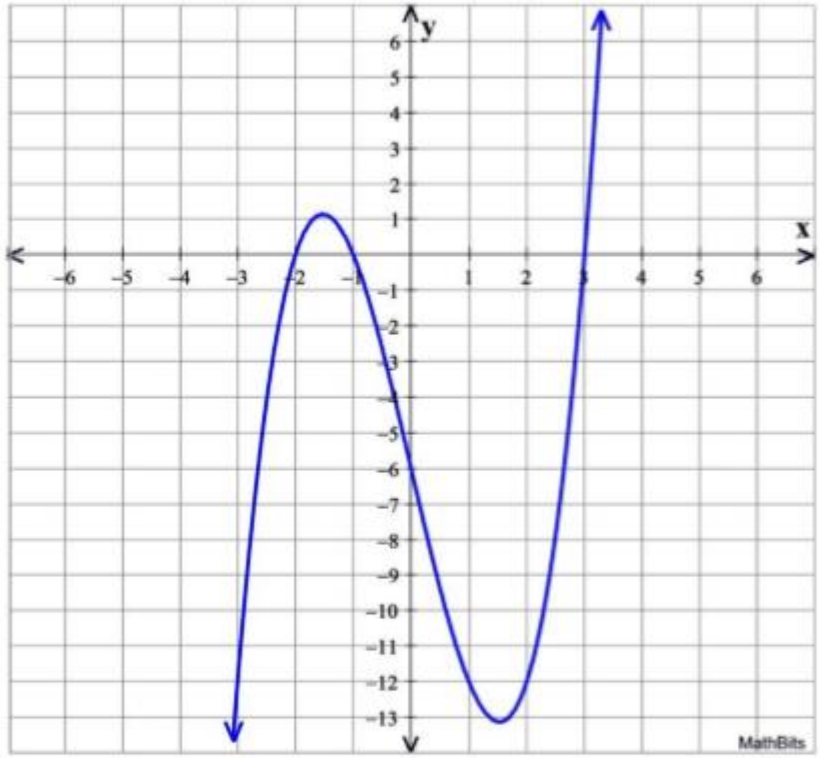
11. $(4g^2 - 9) \div (2g + 3)$

12. $(2x^2 - 5x - 4) \div (x - 3)$

13. $\dfrac{u^3+5u-12}{u-3}$

Questions pertain to the graph of f(x) shown below.

$$f(x) = y = x^3 - 7x - 6$$



1. Find f(3).

2. Find the remainder for: $\dfrac{x^3 - 7x - 6}{x - 3}$

3. Find f(-1).

4. Find the remainder for: $\dfrac{x^3 - 7x - 6}{x + 1}$

5. Find f(1).

6. Find the remainder for: $\dfrac{x^3 - 7x - 6}{x - 1}$

7. Find the value of f(-3).

8. Find the remainder for: $\dfrac{x^3 - 7x - 6}{x + 3}$

Describe why when dividing a polynomial by "x – a", the remainder can be found by examining the graph of the polynomial.

9. Using the graph, determine the remainder for: $\dfrac{x^3 - 7x - 6}{x - 1}$

10. Using the graph, determine the remainder for: $\dfrac{x^3 - 7x - 6}{x}$

Module 1 – Polynomial, Rational, and Radical Relationships

The following questions are to be completed and ALL work must be shown. You may use your calculator.

1. What are the zeros of the polynomial function $y = (x - 3)(2x + 1)(x - 1)$?

(A) $\frac{1}{2}, 1, 3$ (B) $-1, 1, 3$ (C) $-\frac{1}{2}, 1, 3$ (D) $-3, \frac{1}{2}, -1$

2. What is the factored form of $2x^3 + 5x^2 - 12x$?

(F) $(x + 4)(2x - 3)$

(H) $x(x + 4)(2x - 3)$

(G) $(x - 4)(2x + 3)$

(I) $x(x - 4)(2x + 3)$

3. Which is the cubic polynomial in standard form with roots 3, −6, and 0?

(A) $x^2 - 3x - 18$

(C) $x^3 - 3x^2 - 18x$

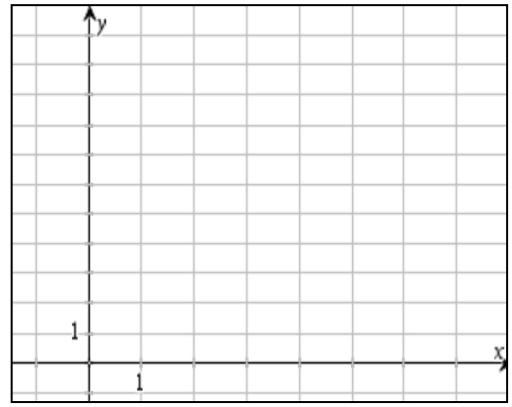
(B) $x^2 + 3x - 18$

(D) $x^3 + 3x^2 - 18x$

4. What is the degree of the polynomial $5x + 4x^2 + 3x^3 - 5x$?

(A) 1 (B) 2 (C) 3 (D) 4

5. Use the equation $y = 2(x - 3)^2 + 5$ to answer the following questions:



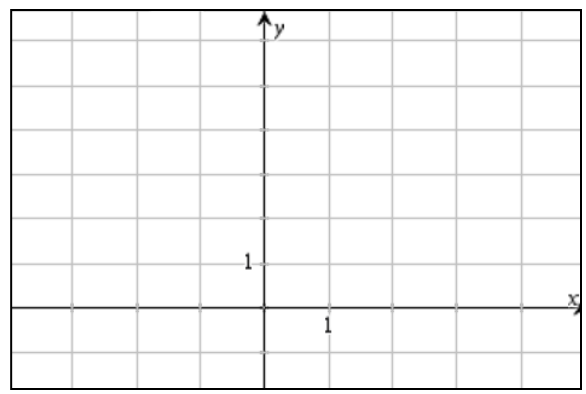
(a) Make an accurate sketch on the grid.

(b) What are the coordinates of the vertex?

(c) What is the equation of the axis of symmetry?

(d) Describe the roots of the quadratic equation.

6. Solve the system $\begin{cases} 2x + y = 4 \\ x = y - 1 \end{cases}$ algebraically and graphically.



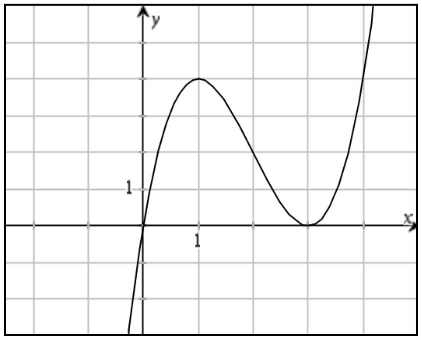
7. Use your calculator to find the zeros of $x^3 + x^2 - 17x + 15 = 0$. Write your answers on the right. Do not draw a graph.

Zeros:

$x =$ $x =$ $x =$

8. Simplify $\left(9x^3 - 4x + 2\right) - \left(x^3 + 3x^2 + 1\right)$. Then name the polynomial by degree and the number of terms.

1. Write an equation in standard form for the polynomial shown in the graph.



a) zeros

b) factored form

c) standard form

2. Write each polynomial in factored form. **<u>Factor completely</u>**.

a) $y = x^3 - 49x$

b) $y = x^3 - 7x^2 + 12x$

3. Write the equation for a polynomial function whose zeros are $x = -5$, $x = 1$, and $x = 2$ in

a) factored form

b) standard form

4. Find the zeros for each of the following functions and give the degree of the function:

a) $y = x(x+1)(x-4)$

zeros:

degree:

b) $y = (x+6)^2(x-1)$

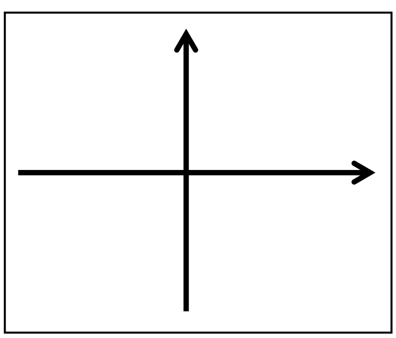
zeros:

degree:

Extra for Experts

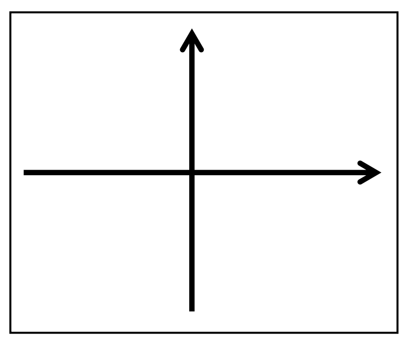
1. (a) What is a quadratic polynomial function with zeros 3 and -3 ?

Make a sketch of the function you found in part (a).



(b) What is a cubic polynomial function with zeros 3, 3, and -3?

Make a sketch of the function you found in part (b).



(c)Look at your graphs. How do the graphs differ? How are they similar?

Even and Odd Functions

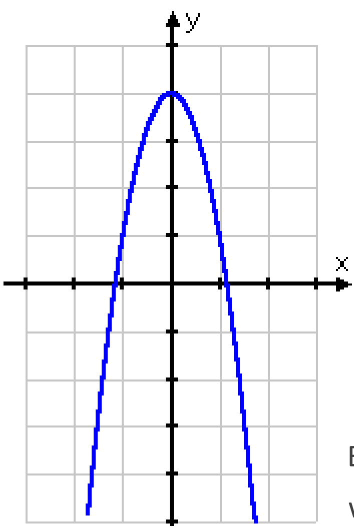
You may be asked to "determine algebraically" whether a function is even or odd. To do this, you take the function and plug $-x$ in for x, and then simplify. If you end up with the exact same function that you started with (that is, if $f(-x) = f(x)$, so all of the signs are the same), then the function is even. If you end up with the exact opposite of what you started with (that is, if $f(-x) = -f(x)$, so all of the signs are switched), then the function is odd.

In all other cases, the function is "neither even nor odd".

Let's see what this looks like in action:

- **Determine algebraically whether $f(x) = -3x^2 + 4$ is even, odd, or neither.**

If I graph this, I will see that this is "**symmetric**" about the y-axis"; in other words, whatever the graph is doing on one side of the y-axis is mirrored on the other side:



This mirroring about the y-axis is a hallmark of even functions.

Also, I note that the exponents on all of the terms are even — the exponent on the constant term being zero: $4x^0 = 4 \times 1 = 4$. These are helpful clues that strongly suggest to me that I've got an even function here.

But the question asks me to make the determination *algebraically*, which means that I need to do the algebra.

So I'll plug $-x$ in for x, and simplify:

$f(-x) = -3(-x)^2 + 4$

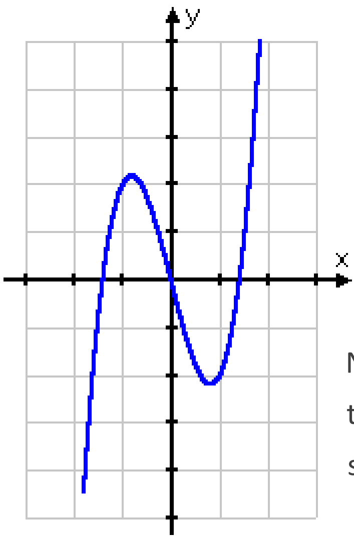
$= -3(x^2) + 4$

$= -3x^2 + 4$

I can see, by comparing the original function with my final result above, that I've got a match, which means that: **$f(x)$ is even**

- **Determine algebraically whether $f(x) = 2x^3 - 4x$ is even, odd, or neither.**

If I graph this, I will see that it is "**symmetric**" about the origin"; that is, if I start at a point on the graph on one side of the *y*-axis, and draw a line from that point through the origin and extending the same length on the other side of the *y*-axis, I will get to another point on the graph.



You can also think of this as the half of the graph on one side of the *y*-axis is the upside-down version of the half of the graph on the other side of the *y*-axis. This symmetry is a hallmark of odd functions.

Note also that all the exponents in the function's rule are odd, since the second term can be written as $4x = 4x^1$. This is a useful clue. I should expect this function to be odd.

The question asks me to make the determination algebraically, so I'll plug $-x$ in for x, and simplify:

$f(-x) = 2(-x)^3 - 4(-x)$

$= 2(-x^3) + 4x$

$= -2x^3 + 4x$

For the given function to be odd, I need the above result to have all opposite signs from the original function. So I'll write the original function, and then switch all the signs:

original: $f(x) = 2(x)^3 - 4(x)$

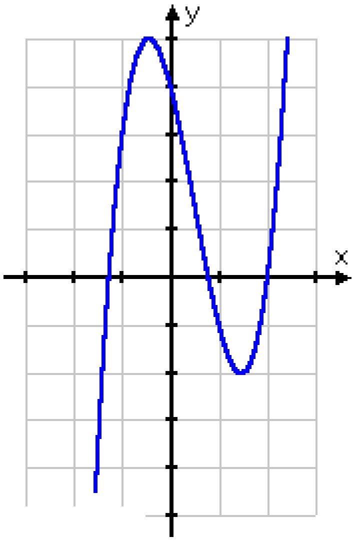
switched: $-f(x) = -2x^3 + 4x$

Comparing this to what I got, I see that they're a match. When I plugged $-x$ in for x, all the signs switched. This means that, as I'd expected: **$f(x)$ is odd.**

- **Determine algebraically whether $f(x) = 2x^3 - 3x^2 - 4x + 4$ is even, odd, or neither.**

This function is the sum of the previous two functions. But, while the sum of an odd and an even number is an odd number, I cannot conclude the same of the sum of an odd and an even function.

Note that the graph of this function does not have the symmetry of either of the previous ones:



...nor are all of its exponents either even or odd.

Based on the exponents, as well as the graph, I would expect this function to be *neither* even *nor* odd. To be sure, though (and in order to get full credit for my answer), I'll need to do the algebra.

I'll plug $-x$ in for x, and simplify:

$$\begin{aligned} f(-x) &= 2(-x)^3 - 3(-x)^2 - 4(-x) + 4 \\ &= 2(-x^3) - 3(x^2) + 4x + 4 \\ &= -2x^3 - 3x^2 + 4x + 4 \end{aligned}$$

I can see, by a quick comparison, that this does not match what I'd started with, so this function is not even. What about odd?

To check, I'll write down the exact opposite of what I started with, being the original function, but with all of the signs changed:

$-f(x) = -2x^3 + 3x^2 + 4x - 4$

This doesn't match what I came up with, either. So the original function isn't odd, either. Then, as I'd expected:

f **(*x*) is neither even nor odd.**

As you can see, the sum or difference of an even and an odd function is *not* an odd function. In fact, you'll discover that the sum or difference of two even functions is another even function, but the sum or difference of two odd functions is another odd function.

There is (exactly) one function that is both even and odd; it is the zero function, $f(x) = 0$.

Module 1 – Polynomial, Rational, and Radical Relationships

Answer the questions below carefully. Be sure to read carefully!

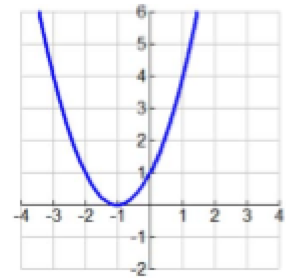
1. Which of the following functions is even?

A) $f(x) = x^2 + x$ B) $f(x) = x^3 + x^2$

C) $f(x) = x^4 + x^2$ D) $f(x) = (x + 1)^2$

2. Regarding the graph at the right, the parabola is:

A) even since it is symmetric
B) odd since it is symmetric
C) not a function since it fails the horizontal line test
D) neither even nor odd



3. Is the parent absolute value function, $f(x) = |x|$, odd, eve, or neither? Explain.

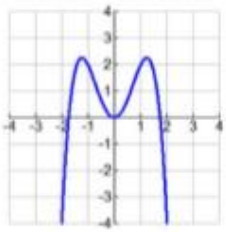
4. If $f(x) = x^3 + x$, which of the following statements must be true? (Choose all that apply.)

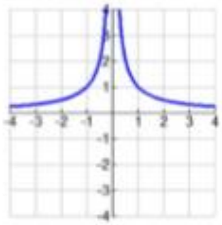
A) f(x) is odd B) f(x) is even C) f(x) is neither
D) f(x) = f(-x) E) f(-x) = -f(x)
F) f(x) is symmetric about the origin G) f(x) is symmetric about the origin

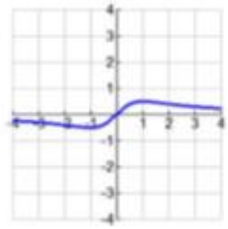
5. Given the functions shown below, determine which of the functions are odd, even or neither. Show your algebraic work to confirm your answers.

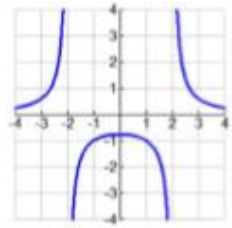
1) $f(x) = 4x^3 – 9$
2) $f(x) = x^2 + 4x + 4$
3) $f(x) = x^5 + 4x^3 – 2x$
4) $f(x) = |x| + 2$
5) $f(x) = \frac{x^2+1}{3x^3+x}$
6) $f(x) = \frac{x^2}{x^3-x}$

6. Which of the following functions is odd? Explain your answer.

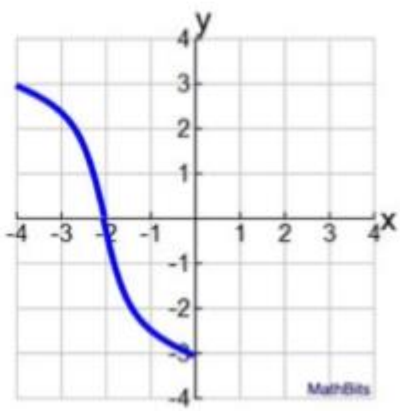
a) 

b) 

c) 

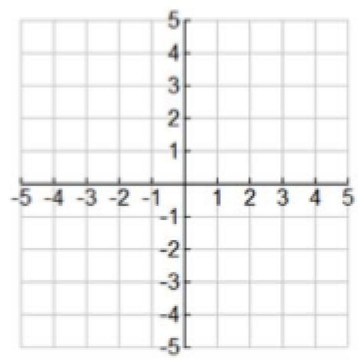
d) 

7. The graph shown below is a portion an even function on the interval {-4, 4} Complete the graph on the given interval.



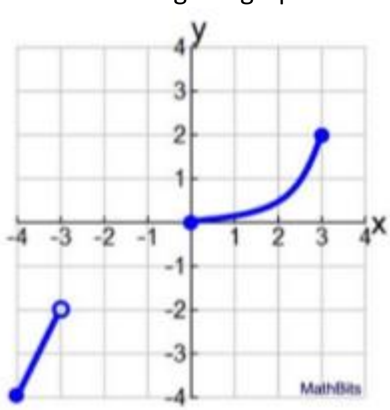
8. Sketch the graph of an odd function of that has the following properties. There is more than one correct answer.

Domain: [-5, 5] Range: [-2, 2]
Increasing on the interval (-3, 3)
Decreasing on intervals (-5, -3) and (3, 5)

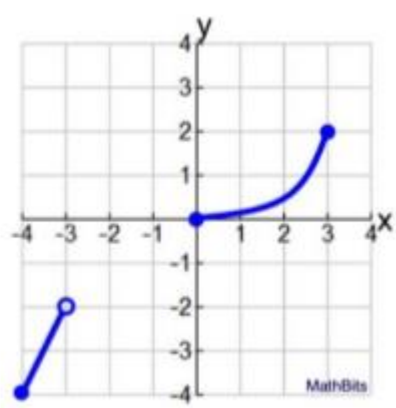


9. The graphs shown below is a portion of a function on the interval [-4, 4].

a) Complete the graph on the given interval assuming the graph to be even.



b) Complete the graph on the given Interval assuming the graph to be odd.



Simplifying Radicals

In 1-28, simplify each radical expression. (All variables represent positive numbers.)

1.	$\sqrt{75}$	2.	$\sqrt{32}$	3.	$\sqrt{45}$	4.	$-\sqrt{300}$
5.	$\sqrt{24}$	6.	$\sqrt[3]{24}$	7.	$\sqrt{54}$	8.	$\sqrt[3]{54}$
9.	$2\sqrt{50}$	10.	$-3\sqrt{28}$	11.	$5\sqrt{12}$	12.	$3\sqrt[3]{56}$
13.	$\frac{1}{2}\sqrt{8}$	14.	$\frac{2}{5}\sqrt{500}$	15.	$-\frac{2}{3}\sqrt{27}$	16.	$\frac{5}{8}\sqrt{80}$
17.	$\sqrt{49x}$	18.	$\sqrt{a^2b}$	19.	$\sqrt{n^3}$	20.	$\sqrt{4cd^2}$
21.	$\sqrt{9y^9}$	22.	$7\sqrt{40x^2}$	23.	$\sqrt{200w}$	24.	$\sqrt{18a^3b^5}$
25.	$-2\sqrt{63k^4}$	26.	$\frac{3}{2}\sqrt{20y}$	27.	$\sqrt[3]{16x^6}$	28.	$\frac{4}{5}\sqrt[3]{125y^5}$

Adding and Subtracting Radicals

In 1-15, combine the radicals. (All variables represent positive numbers.)

1.	$8\sqrt{3}+9\sqrt{3}$	2.	$6\sqrt{7}-4\sqrt{7}$	3.	$8\sqrt{6}-\sqrt{6}$
4.	$\sqrt{72}+\sqrt{18}$	5.	$\sqrt{48}+\sqrt{75}$	6.	$\sqrt{63}-\sqrt{28}$
7.	$\sqrt{180}-\sqrt{80}$	8.	$2\sqrt{8}-\sqrt{32}$	9.	$\sqrt{54}+3\sqrt{24}$
10.	$\sqrt{81x}+\sqrt{9x}$	11.	$\sqrt{45y}-\sqrt{20y}$	12.	$9\sqrt{x^3}-\sqrt{9x^3}$

In 13-20, combine the radicals. (All variables represent positive numbers.)

13.	$\sqrt{700}+8\sqrt{7}-3\sqrt{28}$	14.	$\sqrt{160}-\sqrt{40}+\sqrt{90}$
15.	$\sqrt{50}-\sqrt{98}+\sqrt{128}$	16.	$\sqrt{192}-\sqrt{27}+\sqrt{108}$
17.	$\sqrt{20}+\sqrt{5}+\sqrt{150}-\sqrt{96}$	18.	$\sqrt{125}+\sqrt{12}-\sqrt{45}+\sqrt{75}$
19.	$\sqrt[3]{64x}-\sqrt[3]{8x}+\sqrt[3]{27x}$	20.	$\sqrt[3]{250y^3}-\sqrt[3]{16y^3}$

Multiplying Radicals

In #1-28, multiply and express each product in simplest form. (All variables represent positive numbers.)

1. $\sqrt{5} \bullet \sqrt{20}$
2. $3\sqrt{32} \bullet \sqrt{2}$
3. $4\sqrt{5} \bullet \sqrt{10}$
4. $\frac{1}{3}\sqrt{18} \bullet \sqrt{6}$
5. $8\sqrt{3} \bullet \frac{1}{2}\sqrt{15}$
6. $2\sqrt{14} \bullet 6\sqrt{7}$
7. $\sqrt{\frac{1}{2}} \bullet \sqrt{72}$
8. $\sqrt{\frac{3}{4}} \bullet \sqrt{3}$
9. $5\sqrt{\frac{1}{3}} \bullet \sqrt{18}$
10. $\sqrt{x} \bullet \sqrt{x^5}$
11. $\sqrt{6y^3} \bullet \sqrt{2y}$
12. $\sqrt{ab^3} \bullet \sqrt{ab^5}$
13. $\sqrt[3]{5} \bullet \sqrt[3]{25}$
14. $2\sqrt[3]{2} \bullet 4\sqrt[3]{4}$
15. $\sqrt[3]{9x^2} \bullet \sqrt[3]{6x}$
16. $\sqrt{3}\left(4\sqrt{3}+\sqrt{12}\right)$
17. $3\sqrt{2}\left(2\sqrt{8}-\sqrt{3}\right)$
18. $2\sqrt{5}\left(3\sqrt{2}+\sqrt{45}-4\sqrt{5}\right)$
19. $\left(2+\sqrt{7}\right)\left(3+\sqrt{7}\right)$
20. $\left(9-\sqrt{2}\right)\left(7+\sqrt{2}\right)$
21. $\left(5+\sqrt{3}\right)^2$
22. $\left(6-\sqrt{3}\right)^2$
23. $\left(10-\sqrt{5}\right)\left(10+\sqrt{5}\right)$
24. $\left(3+2\sqrt{3}\right)\left(3-2\sqrt{3}\right)$
25. $\left(\sqrt{17}\right)^2$
26. $\left(2\sqrt{10}\right)^2$
27. $\left(3\sqrt{7}\right)^2$
28. $\left(\sqrt[3]{2}\right)^3$

Dividing Radicals

In #1-20, divide and express each quotient in simplest form. (All variables represent positive numbers.)

1. $\sqrt{80} \div \sqrt{5}$
2. $12\sqrt{7} \div 2\sqrt{7}$
3. $\sqrt{170} \div \sqrt{17}$
4. $\sqrt{150} \div \sqrt{3}$
5. $10\sqrt{24} \div \sqrt{2}$
6. $9\sqrt{60} \div 3\sqrt{3}$
7. $\frac{28\sqrt{90}}{7\sqrt{2}}$
8. $\frac{24\sqrt{48}}{48\sqrt{6}}$
9. $\frac{45\sqrt{40}}{60\sqrt{10}}$
10. $\frac{30\sqrt[3]{128}}{6\sqrt[3]{2}}$
11. $\frac{6\sqrt[3]{60}}{15\sqrt[3]{10}}$
12. $\frac{3\sqrt[3]{96}}{12\sqrt[3]{4}}$
13. $\frac{\sqrt{98x^3}}{\sqrt{2x}}$
14. $\frac{2\sqrt{54y^5}}{6\sqrt{2y}}$
15. $\frac{\sqrt{20ab^5}}{2\sqrt{5ab^3}}$
16. $\frac{\sqrt{75}+\sqrt{48}}{\sqrt{3}}$
17. $\frac{2\sqrt{5}+6\sqrt{15}}{2\sqrt{5}}$
18. $\frac{\sqrt{108}-\sqrt{150}}{\sqrt{6}}$
19. $\frac{\sqrt{375}+\sqrt{540}}{\sqrt{3}}$
20. $\frac{\sqrt{180}-\sqrt{125}}{\sqrt{5}}$

Module 1 – Polynomial, Rational, and Radical Relationships

Working with Radicals

Match the radical expressions on the left with an equivalent expression on the right.

1. _____	$\sqrt{24x^3y^4}$	**A.**	$10x^2\sqrt[3]{2x}$
2. _____	$\sqrt[3]{54x^4}$	**B.**	$4x\sqrt{x}$
3. _____	$\sqrt{3xy}\bullet\sqrt{8xy^3}$	**C.**	$3x\sqrt[3]{2x}$
4. _____	$\sqrt[3]{5x^3}\bullet 2\sqrt[3]{50x^4}$	**D.**	$2x^2$
5. _____	$\sqrt{32x^4}\div\sqrt{2x}$	**E.**	$6x\sqrt[3]{2}$
6. _____	$2\sqrt[3]{2xy^2}\bullet\sqrt[3]{32x^2y^5}$	**F.**	$2xy^2\sqrt{6}$
7. _____	$\sqrt{2x^3}\left(\sqrt{50x}-\sqrt{32x}\right)$	**G.**	$2xy\sqrt{2}$
8. _____	$\sqrt[3]{8x^8}\bullet\sqrt[3]{4x^8}$	**H.**	$2xy^2\sqrt{6x}$
9. _____	$9\sqrt[3]{48x^7}\div 3\sqrt[3]{3x^4}$	**I.**	$8xy^2\sqrt[3]{y}$
10. _____	$\sqrt{56x^5y^5}\div\sqrt{7x^3y^3}$	**J.**	$2x^5\sqrt[3]{4x}$

Rational Exponents and Radicals

Definition of $b^{\frac{1}{n}}$	For any real number b and any positive integer n, $b^{\frac{1}{n}} = \sqrt[n]{b}$, except when $b < 0$ and n is even.
Definition of $b^{\frac{m}{n}}$	For any nonzero real number b, and any integers m and n, with $n > 1$, $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$, except when $b < 0$ and n is even.

Example 1: Write $28^{\frac{1}{2}}$ in radical form.

Notice that 28 > 0.

$28^{\frac{1}{2}} = \sqrt{28}$

$= \sqrt{2^2 \cdot 7}$

$= \sqrt{2^2} \cdot \sqrt{7}$

$= 2\sqrt{7}$

Example 2: Evaluate $\left(\frac{-8}{-125}\right)^{\frac{1}{3}}$.

Notice that –8 < 0, –125 < 0, and 3 is odd.

$\left(\frac{-8}{-125}\right)^{\frac{1}{3}} = \frac{\sqrt[3]{-8}}{\sqrt[3]{-125}}$

$= \frac{-2}{-5}$

$= \frac{2}{5}$

Exercises

Write each expression in radical form, or write each radical in exponential form.

1. $11^{\frac{1}{7}}$

2. $15^{\frac{1}{3}}$

3. $300^{\frac{3}{2}}$

4. $\sqrt{47}$

5. $\sqrt[3]{3a^5b^2}$

6. $\sqrt[4]{162p^5}$

Evaluate each expression.

7. $-27^{\frac{2}{3}}$

8. $216^{\frac{1}{3}}$

9. $(0.0004)^{\frac{1}{2}}$

Write each expression in radical form, or write each radical in exponential form.

1. $3^{\frac{1}{6}}$

2. $8^{\frac{1}{5}}$

3. $\sqrt{51}$

4. $\sqrt[4]{15^3}$

5. $12^{\frac{2}{3}}$

6. $\sqrt[3]{37}$

7. $(c^3)^{\frac{3}{5}}$

8. $\sqrt[3]{6xy^2}$

Evaluate each expression.

9. $32^{\frac{1}{5}}$

10. $81^{\frac{1}{4}}$

11. $27^{\frac{1}{3}}$

12. $4^{\frac{1}{2}}$

13. $16^{\frac{3}{2}}$

14. $(-243)^{\frac{4}{5}}$

15. $27^{\frac{1}{3}} \cdot 27^{\frac{5}{3}}$

16. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$

Rationalize the denominator.

1. $\sqrt{\dfrac{1}{3}}$

2. $\sqrt{\dfrac{2}{5}}$

3. $\dfrac{6}{\sqrt{3}}$

4. $\dfrac{2\sqrt{6}}{\sqrt{10}}$

5. $\dfrac{9-2\sqrt{3}}{\sqrt{3}+6}$

6. $\dfrac{6}{\sqrt{3}-\sqrt{2}}$

7. $\dfrac{5+\sqrt{3}}{4+\sqrt{3}}$

8. $\dfrac{5}{8-\sqrt{6}}$

Radical Equations

Solve each equation. Check your answer.

1.	$\sqrt{x-4}+6=10$	2.	$\sqrt{x+13}-8=-2$
3.	$8-\sqrt{x+12}=3$	4.	$\sqrt{x-8}+5=7$
5.	$\sqrt{y}-7=0$	6.	$\sqrt[3]{n+8}-6=-3$
7.	$5+\sqrt{4y-5}=12$	8.	$\sqrt{2t-7}=\sqrt{t+2}$

Solve each equation. Check your answer.

1.	$\sqrt{2x+5}-4=3$	2.	$6+\sqrt{3x+1}=11$
3.	$\sqrt{x+6}=5-\sqrt{x+1}$	4.	$\sqrt{x-3}=\sqrt{x+4}-1$
5.	$\sqrt{x-15}=3-\sqrt{x}$	6.	$\sqrt{x-10}=1-\sqrt{x}$
7.	$6+\sqrt{4x+8}=9$	8.	$2+\sqrt{3y+5}=10$
9.	$\sqrt{x-4}=\sqrt{2x-13}$	10.	$\sqrt{7a-2}=\sqrt{a+3}$
11.	$\sqrt{x+5}-\sqrt{x}=-2$	12.	$\sqrt{b-6}+\sqrt{b}=3$

Rational Expressions

State the excluded values for each.

1. $\frac{60x^3}{12x}$

2. $\frac{70v^2}{100v}$

3. $\frac{m+7}{m^2+4m-21}$

4. $\frac{n^2+6n+5}{n+1}$

5. $\frac{35x-35}{25x-40}$

6. $\frac{-n^2+16n-63}{n^2-2n-35}$

Simplify each and state the excluded values.

1. $\frac{p+4}{p^2+6p+8}$

2. $\frac{9}{15a-15}$

3. $\frac{2a^2+10a}{3a^2+15a}$

4. $\frac{p^2-3p-10}{p^2+p-2}$

5. $\frac{x^2+x-6}{x^2+8x+15}$

6. $\frac{a^2+5a+4}{a^2+9a+20}$

7. $\frac{x^2-2x-15}{x^2-6x+5}$

8. $\frac{10x-6}{10x-6}$

Multiplying and Dividing Rational Expressions

1. $\frac{-2u^3y}{15xz^5} \cdot \frac{25x^3}{14u^2y^2}$

2. $\frac{a+y}{6} \cdot \frac{4}{y+a}$

3. $\frac{n^5}{n-6} \cdot \frac{n^2-6n}{n^3}$

4. $\frac{a-y}{w+n} \cdot \frac{w^2-n^2}{y-a}$

5. $\frac{x^2-5x-24}{6x+2x^2} \cdot \frac{5x^2}{8-x}$

6. $\frac{x-5}{10x-2} \cdot \frac{25x^2-1}{x^2-10x+25}$

7. $\frac{a^5y^3}{wy^7} \div \frac{a^3w^2}{w^5y^2}$

8. $\left(\frac{2xy}{w^2}\right)^3 \div \frac{24x^2}{w^5}$

9. $\frac{x+y}{6} \div \frac{x^2-y^2}{3}$

10. $\frac{3x+6}{x^2-9} \div \frac{6x^2+12x}{4x+12}$

11. $\frac{2s^2-7s-15}{(s+4)^2} \div \frac{s^2-10s+25}{s+4}$

12. $\frac{9-a^2}{a^2+5a+6} \div \frac{2a-6}{5a+10}$

13. $\frac{5r^2}{r^2-4} \cdot \frac{r+2}{10r^5}$

14. $\frac{7g}{y^2} \div 21g^3$

15. $\frac{80y^4}{49z^5v^7} \div \frac{25y^5}{14z^{12}v^5}$

16. $\frac{3x^2}{x+2} \div \frac{3x}{x^2-4}$

17. $\frac{q^2+2q}{6q} \div \frac{q^2-4}{3q^2}$

18. $\frac{w^2-5w-24}{w+1} \cdot \frac{w^2-6w-7}{w+3}$

19. $\frac{t^2+19t+84}{4t-4} \cdot \frac{2t-2}{t^2+9t+14}$

20. $\frac{x^2-5x+4}{2x-8} \div (3x^2-3x)$

21. $\frac{16a^2+40a+25}{3a^2-10a-8} \div \frac{4a+5}{a^2-8a+16}$

22. $\frac{\frac{c^2y}{2d^2}}{\frac{-c^6}{5d}}$

23. $\frac{\frac{a^2-b^2}{4a}}{\frac{a+b}{2a}}$

24. $\frac{\frac{2x+1}{x}}{\frac{4-x}{x}}$

25. $\frac{\frac{x^2-9}{4}}{\frac{3-x}{8}}$

Adding and Subtracting Rational Expressions

Simplify each expression.

1. $\frac{3}{x}+\frac{5}{y}$

2. $\frac{3}{8p^2r}+\frac{5}{4p^2r}$

3. $\frac{2c-7}{3}+4$

4. $\frac{2}{m^2p}+\frac{5}{p}$

5. $\frac{12}{5y^2}-\frac{2}{5yz}$

6. $\frac{7}{4gh}+\frac{3}{4h^2}$

7. $\frac{2}{a+2}-\frac{3}{2a}$

8. $\frac{5}{3b+d}-\frac{2}{3bd}$

9. $\frac{3}{w-3}-\frac{2}{w^2-9}$

10. $\frac{3t}{2-x}+\frac{5}{x-2}$

11. $\frac{k}{k-n}-\frac{k}{n-k}$

12. $\frac{4z}{z-4}+\frac{z+4}{z+1}$

13. $\frac{1}{x^2+2x+1}+\frac{x}{x+1}$

14. $\frac{2x+1}{x-5}-\frac{4}{x^2-3x-10}$

15. $\frac{n}{n-3}+\frac{2n+2}{n^2-2n-3}$

16. $\frac{3}{y^2+y-12}-\frac{2}{y^2+6y+8}$

Simplify each expression.

1. $\frac{5}{6ab}-\frac{7}{8a}$

2. $\frac{5}{12x^4y}-\frac{1}{5x^2y^3}$

3. $\frac{1}{6c^2d}+\frac{3}{4cd^3}$

4. $\frac{4m}{3mn}+2$

5. $2x-5-\frac{x-8}{x+4}$

6. $\frac{4}{a-3}+\frac{9}{a-5}$

7. $\frac{16}{x^2-16}+\frac{2}{x+4}$

8. $\frac{2-5m}{m-9}+\frac{4m-5}{9-m}$

9. $\frac{y-5}{y^2-3y-10}+\frac{y}{y^2+y-2}$

10. $\frac{5}{2x-12}-\frac{20}{x^2-4x-12}$

11. $\frac{2p-3}{p^2-5p+6}-\frac{5}{p^2-9}$

12. $\frac{1}{5n}-\frac{3}{4}+\frac{7}{10n}$

13. $\frac{2a}{a-3}-\frac{2a}{a+3}+\frac{36}{a^2-9}$

14. $\frac{\frac{2}{x-y}+\frac{1}{x+y}}{\frac{1}{x-y}}$

15. $\frac{\frac{r+6}{r}-\frac{1}{r+2}}{\frac{r^2+4r+3}{r^2+2r}}$

Solving Rational Equations

Solve each equation. Check your solution.

1. $\frac{x}{x-1} = \frac{1}{2}$

2. $2 = \frac{4}{n} + \frac{1}{3}$

3. $\frac{9}{3x} = \frac{-6}{2}$

4. $3 - z = \frac{2}{z}$

5. $\frac{2}{d+1} = \frac{1}{d-2}$

6. $\frac{r-3}{5} = \frac{8}{r}$

7. $\frac{2x+3}{x+1} = \frac{3}{2}$

8. $\frac{-12}{y} = y - 7$

9. $\frac{15}{x} + \frac{9x-7}{x+2} = 9$

10. $\frac{3b-2}{b+1} = 4 - \frac{b+2}{b-1}$

11. $2 = \frac{5}{2q} + \frac{2q}{q+1}$

12. $8 - \frac{4}{z} = \frac{8z-8}{z+2}$

13. $\frac{1}{n+3} + \frac{5}{n^2-9} = \frac{2}{n-3}$

14. $\frac{1}{w+2} + \frac{1}{w-2} = \frac{4}{w^2-4}$

15. $\frac{x-8}{2x+2} + \frac{x}{2x+2} = \frac{2x-3}{x+1}$

16. $\frac{12p+19}{p^2+7p+12} - \frac{3}{p+3} = \frac{5}{p+4}$

17. $\frac{2f}{f^2-4} + \frac{1}{f-2} = \frac{2}{f+2}$

18. $\frac{8}{t^2-9} + \frac{4}{t+3} = \frac{2}{t-3}$

Work Problems

1. In the movie "Little Big League", Billy Heywood seeks assistance from baseball players in solving a homework problem. The problem states, "Joe can paint a house in three hours, and Sam can paint the same house in five hours. How long does it take for them to do it together?"

2. Every year, the junior and senior classes at Hillcrest High School build a house for the community. If it takes the senior class 24 days to complete a house and 18 days if they work with the junior class, how long would it take the junior class to complete a house if they worked alone?

3. Hailey's father has been painting houses for the past 20 years. As a result he can paint a house 1.5 times faster than Hailey can. They work together on a house painting project, and it takes them 14 hours. How many hours, to the nearest hour, would it have taken Hailey if she had been working alone?

4. It took Anthony and Travis 6 hours to rake leaves together last year. The previous year it took Travis 10 hours to do it alone. How long will it take Anthony if he rakes them by himself this year?

5. Noah and Owen paint houses together. If Noah can paint a particular house in 6 days and Owen can paint the same house in 5 days, how long would it take the two of them if they work together?

6. Working alone, Ryan can dig a 10 ft by 10 ft hole in eight hours. Castel can dig the same hole in six hours. How long would it take them if they worked together?

7. Shawna can pour a large concrete driveway in six hours. Dan can pour the same driveway in seven hours. Find how long it would take them if they worked together.

8. Working together, Paul and Daniel can pick forty bushels of apples in 7 hours. Had he done it alone it would have taken Daniel 12 hours. Find how long it would take Paul to do it alone.

Module 1 – Polynomial, Rational, and Radical Relationships

Systems of Equations

$$x + 5y = 3$$
$$3x - 2y = -8$$

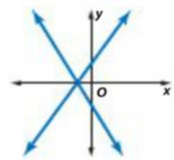
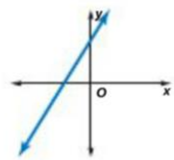
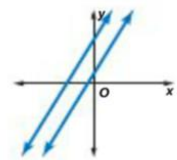
KeyConcept Elimination Method

Step 1 Multiply one or both equations by a number to result in two equations that contain opposite terms.

Step 2 Add the equations, eliminating one variable. Then solve the equation.

Step 3 Substitute to solve for the other variable.

ConceptSummary Characteristics of Linear Systems

Consistent and Independent	Consistent and Dependent	Inconsistent
		
intersecting lines; one solution	same line; infinitely many solutions	parallel lines; no solution

KeyConcept Solving by Substitution

Step 1 When necessary, solve at least one equation for one variable.

Step 2 Substitute the resulting expression from Step 1 into the other equation to replace the variable. Then solve the equation.

Step 3 Substitute the value from Step 2 into either equation, and solve for the other variable. Write the solution as an ordered pair.

Solve the systems of equations and check. Use either the elimination or the substitution method.

1.	$y = 4x$ $x + y = 5$	**2.**	$y = 2x$ $x + 3y = -14$
3.	$y = 3x$ $2x + y = 15$	**4.**	$x = -4y$ $3x + 2y = 20$
5.	$y = x - 1$ $x + y = 3$	**6.**	$x = y - 7$ $x + 8y = 2$
7.	$3x + 4y = 2$ $4x - 4y = 12$	**8.**	$3x - y = -1$ $-3x - y = 5$
9.	$2x - 3y = 9$ $-5x - 3y = 30$	**10.**	$x - y = 4$ $2x + y = -4$
11.	$2x - 3y = 21$ $5x - 2y = 25$	**12.**	$3x + 2y = -26$ $4x - 5y = -4$
13.	$3x - 6y = -3$ $2x + 4y = 30$	**14.**	$5x + 2y = -3$ $3x + 3y = 9$

Module 1 – Polynomial, Rational, and Radical Relationships

Systems in **THREE** variables can have one solution, infinite solutions, or no solution. A solution of such a system is an ordered triple (x, y, z). When you have three variables you are looking for the intersection of planes.

Solve the system of equations.

1. $3x - 2y + 4z = 35$
   $-4x + y - 5z = -36$
   $5x - 3y + 3z = 31$

2. $2x - y + z = -13$
   $x + 2y - z = 6$
   $3x - 2y + 3z = -16$

3. $x - z = 4$
   $x + y = -2$
   $2x + y + z = 0$

4. $x - 2y + 3z = 7$
   $2x + y + z = 4$
   $-3x + 2y - 2z = -10$

5. $x - 4y + z = 18$
   $2x + y - 5z = -21$
   $x + 2y - 2z = -15$

6. $a + b + c = 5$
   $9a + 3b + c = 25$
   $4a - 2b + c = 20$

7. $4x - 5y - 6z = 26$
   $x - 6y + z = 40$
   $5x - 3y - 7z = 14$

Solving a Quadratic – Linear System

Solve the systems of equations algebraically. Check your answer with your calculator.

1. $y = x^2$
   $y = 2x$

2. $y = -2x^2 + 7x - 2$
   $y = 3 - 4x$

3. $y = x^2 + 7x + 12$
   $y = 2x + 8$

4. $y = x^2 - x - 20$
   $y = 3x + 12$

Solving a Circular – Linear System

Solve the systems of equations algebraically. Check your answers.

1. $x^2 + y^2 = 100$
   $y - x = 2$

2. $(x - 3)^2 + (y + 2)^2 = 16$
   $2x + 2y = 10$

SOLVING QUADRATIC EQUATIONS USING THE QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

PROBLEM

Solve the quadratic equation $2x^2 + 3x - 4 = 0$ **using the quadratic formula.**

- Write the equation in standard form: $ax^2 + bx + c = 0$ — $2x^2 + 3x - 4 = 0$
- Write the Quadratic Formula — $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Substitute $a = 2,\ b = 3,\ c = -4$ — $x = \frac{-(3) \pm \sqrt{(3)^2 - 4(2)(-4)}}{2(2)}$
- Use your calculator to simplify — $x = \frac{-3 \pm \sqrt{41}}{4}$
- Write the solutions separately — $x = \frac{-3 + \sqrt{41}}{4},\quad x = \frac{-3 - \sqrt{41}}{4}$

PRACTICE Use the Quadratic Formula to find the solutions of each equation.

1. $x^2 + 6x = 10$
2. $2x^2 = 4x + 3$
3. $x^2 + 4x + 3 = 0$
4. $2x^2 - 3x - 15 = 5$
5. $x^2 + 2x - 1 = 2$
6. $5x^2 = 80$
7. $9x^2 = 4 + 7x$
8. $8x^2 + 7x - 15 = -7$

PERFECT SQUARE TRINOMIALS

$$(x-a)^2 = x^2 + 2ax + a^2$$

PROBLEM

$$x^2 + 8x + 16$$

Look at these patterns in perfect square trinomials

- $\left[\frac{1}{2}(b)\right]^2 = c$ $\qquad$ $\left[\frac{1}{2}(8)\right]^2 = 16$
- $2 \bullet \sqrt{c} = b$ $\qquad$ $2 \bullet \sqrt{16} = 8$

Since this is true we can rewrite $x^2 + 8x + 16$ as $(x+4)^2$

PRACTICE

1. Complete the square for each of the following problems

(a) $x^2 + 22x + \square = (\qquad)^2$

(b) $x^2 - 30x + \square = (\qquad)^2$

(c) $x^2 - 4x + \square = (\qquad)^2$

(d) $x^2 + 12x + \square = (\qquad)^2$

2. Find the value of *k* that would make each problem below a perfect square trinomial.

(a) $x^2 - kx + 25$

(b) $x^2 + kx + 49$

(c) $x^2 + kx + 144$

(d) $x^2 - kx + 1$

SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

One of the ways to solve a quadratic equation is to rewrite the left side of the equation as a perfect square trinomial.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

PROBLEM

Solve the quadratic equation $x^2 - 4x + 2 = 0$ by completing the square.

- Rewrite in the form $x^2 + bx = -c$: $x^2 - 4x = -2$
- Find $\left(\frac{b}{2}\right)^2$ $\left(\frac{-4}{2}\right)^2 = 4$
- Add 4 to each side of the equation $x^2 - 4x + \boxed{4} = -2 + \boxed{4}$
- Factor the perfect square trinomial $(x-2)^2 = 2$
- Find the square root of both sides $(x-2) = \pm\sqrt{2}$
- Solve for x $x = 2 \pm \sqrt{2}$
- Write the solutions separately $x = 2 + \sqrt{2}$ and $x = 2 - \sqrt{2}$

PRACTICE Solve each quadratic equation by completing the square.

1. $x^2 + 6x - 3 = 0$
2. $x^2 + 8x = 11$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $x^2 + 4x - 12 = 0$

2. $x^2 - 8x + 15 = 0$

3. $x^2 + 6x = 7$

4. $x^2 - 2x = 15$

5. $x^2 - 14x + 30 = 6$

6. $x^2 + 12x + 21 = 10$

7. $x^2 - 4x + 1 = 0$

8. $x^2 - 6x + 4 = 0$

9. $x^2 - 8x + 10 = 0$

10. $x^2 - 2x = 5$

11. $2x^2 + 20x = -2$

12. $7b^2 - 14b - 56 = 0$

13. $2n^2 + 12n + 10 = 0$

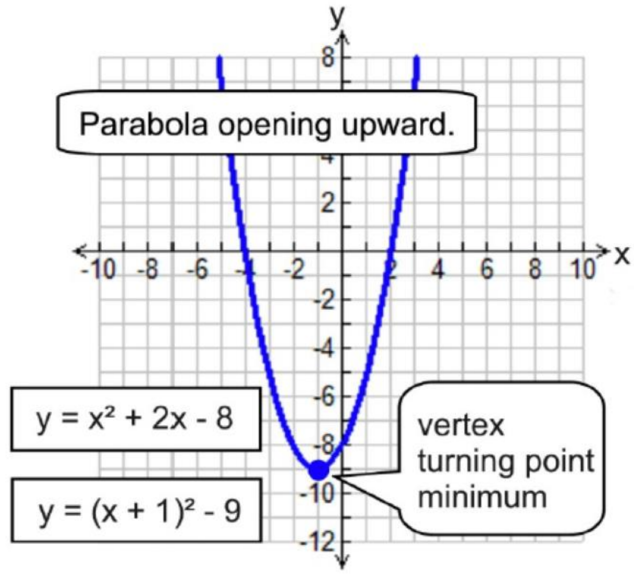
14. $4v^2 - 16v = 65$

15. $2a^2 = -6 + 8a$

16. $6x^2 - 48 = -12x$

Completing the Square to Find the Vertex

Examine the graph at the right. Using the process of completing the square a quadratic function can be expressed in $y = a(x - h)^2 + k$ form. This form is often referred to as "vertex form" since it makes it easy to identify the turning point (maximum or minimum point) of the function.



1. Using the process of completing the square, write the following quadratic function in "vertex form", state the coordinates of the vertex, and indicate the direction of the parabola as concave up or concave down.

(a) $y = x^2 + 4x - 12$

(b) $y = x^2 - 8x + 8$

(c) $y = x^2 + 6x + 7$

2. Watch out for the signs!

$y = -x^2 + 8x - 11$

Completing the Square to Find the Vertex

Write each function in vertex form. State the vertex and whether the function is concave up or down.

1. $y = x^2 - 6x + 3$

2. $y = x^2 + 2x + 7$

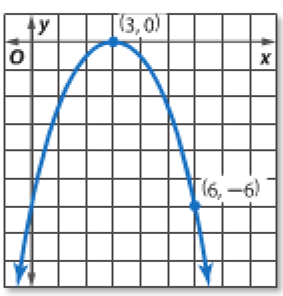
3. $y = 2x^2 - 4x - 3$

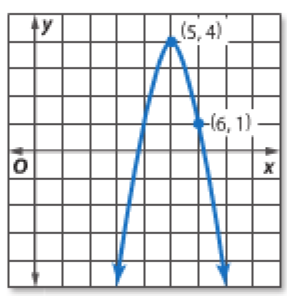
4. $y = x^2 - 4x + 9$

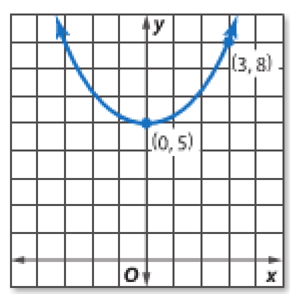
5. $y = -4x^2 - 24x - 15$

6. $Y = -x^2 - 4x + 4$

Write an equation in vertex form for each parabola given below.

1. 

2. 

3. 

Module 1 – Polynomial, Rational, and Radical Relationships

<u>Checkup:</u> Do all work in your notebook for each problem.

Solve these systems algebraically and check.

1. $-x - 5y - 5z = 2$
   $4x - 5y + 4z = 19$
   $X + 5y - z = -20$

2. $4x + 4y + z = 24$
   $2x - 4y + z = 0$
   $5x - 4y - 5z = 12$

3. Working alone, Ryan can dig a 10 ft by 10 ft hole in 5 hours. Castel can dig the same hole in 6 hours. How long would it take if they worked together?

4. Working together, Paul and Daniel can pick 40 bushels of apples in 5 hours. Had he done it alone it would have taken Daniel 9 hours. Find how long it would take Paul to do it alone.

5. Solve this quadratic equation using the Quadratic Formula.

   $2m^2 + 2m - 12 = 0$

Solve these quadratic equations by completing the square.

6. $p^2 + 14p - 38 = 0$
7. $r^2 - 4r - 91 = 7$
8. $5k^2 = 60 - 20k$
9. $2x^2 = 6 + 4x$

Write each function in vertex form. State the vertex and whether the function is concave up or concave down.

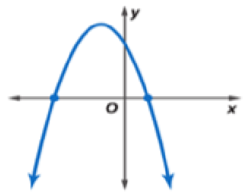
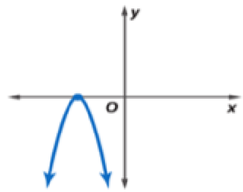
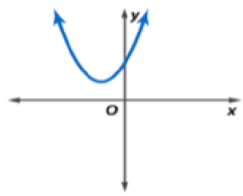
10. $y = x^2 - 10x + 21$
11. $y = 4x^2 - 16x + 16$

Write the equation in vertex form when given the information below for each parabola.

12. Vertex (3, 1) Point (5, 5)
13. Vertex (-4, 3) Point (-3, 1)

KeyConcept Discriminant

Consider $ax^2 + bx + c = 0$, where a, b, and c are rational numbers and $a \neq 0$.

Value of Discriminant	Type and Number of Roots	Example of Graph of Related Function
$b^2 - 4ac > 0$; $b^2 - 4ac$ is a perfect square.	2 real, rational roots	
$b^2 - 4ac > 0$; $b^2 - 4ac$ is *not* a perfect square.	2 real, irrational roots	
$b^2 - 4ac = 0$	1 real rational root	
$b^2 - 4ac < 0$	2 complex roots	

1. Complete the table below.

Equation	$b^2 - 4ac$	Nature of Roots	Sketch
$x^2 - 5x + 4 = 0$			
$-3x^2 + 5x - 4 = 0$			
$4x^2 + 4x + 1 = 0$			
$x^2 - 3x + 1 = 0$			

2. Use the method of your choice to solve the following quadratic equations. Each method can only be used once. Choose from: factoring, completing the square or the quadratic formula. Then use the discriminant to verify your answer.

(a) $4x^2 + x - 3 = 0$ Discriminant

(b) $x^2 - 5x - 9 = 0$ Discriminant

(c) $x^2 - 12x = -16$ Discriminant

METHODS FOR SOLVING QUADRATIC EQUATIONS

FACTORING	use when a quadratic equation can be factored easily
FINDING SQUARE ROOTS	use when solving an equation that can be written in the form $x^2 = d$
COMPLETING THE SQUARE	can be used for any quadratic equation $ax^2 + bx + c = 0$ but is simplest to apply when $a = 1$ and *b* is an even number.
QUADRATIC FORMULA	can be used for any quadratic equation
GRAPHING	can be used for any quadratic equation with integer or irrational solutions

1. Tell what method you would use to solve the quadratic equation. Explain your choice.

   a) $6x^2 - 11x + 7 = 0$

   b) $4x^2 - 36 = 0$

   c) $x^2 + 8x = 9$

2. Use the quadratic formula to solve $2x^2 - 5 = 3x$. Write out the formula and show all work.

3. Solve the equation $x^2 - 6x = -2$ using the method of completing the square. Use your calculator to verify your solution(s). Make a sketch in the space provided on the right.

4. Use the discriminant [$b^2 - 4ac$] to tell whether the equations below have two solutions, one solution, or no solution. Show your work and check your answer with a graph of the equation.

   a) $x^2 + 2x = 1$

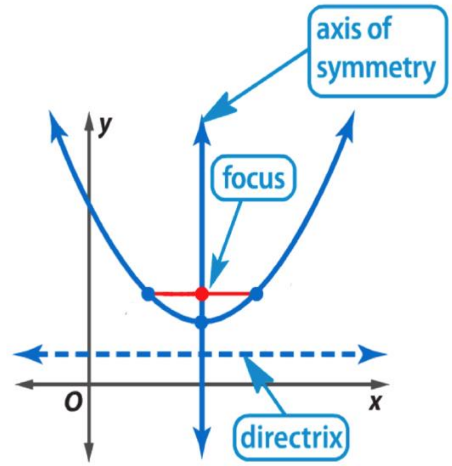
   b) $3x^2 + 7x = -5$

   c) $5x^2 - 6 = 0$

   d) $-x^2 - 6x = 9$

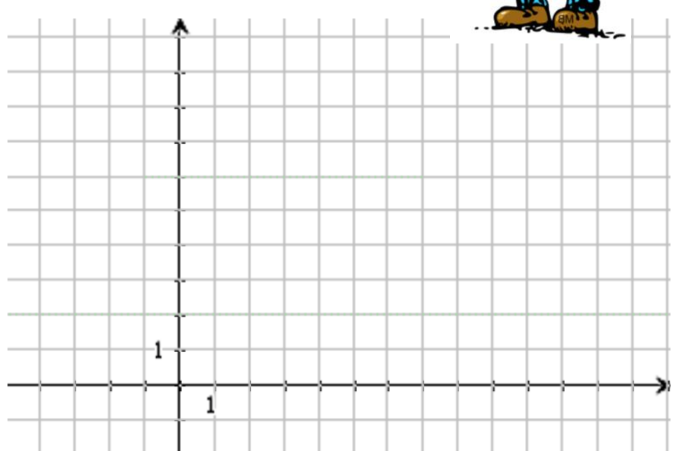
A **PARABOLA** can be defined as a set of all points that are the same distance from a given point called the **FOCUS** and a given line called a **DIRECTRIX**.

The **STANDARD FORM** of this equation with **VERTEX** (h,k) and **AXIS OF SYMMETRY** $x=h$ is $y=\frac{1}{4p}(x-h)^2+k$



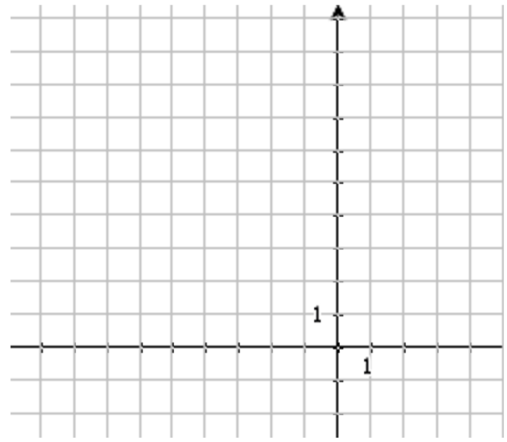
Example #1

Given the equation $y=\frac{1}{8}(x-3)^2+4$ find:



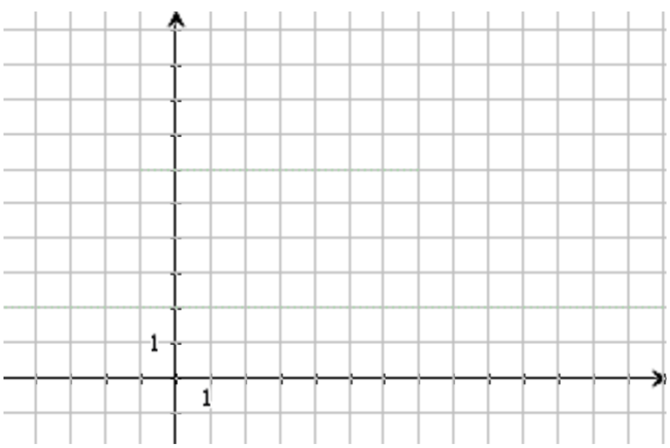
Example #2

Write an equation of a parabola in **STANDARD FORM** that has a **FOCUS** at $(-3,2)$ and a **DIRECTRIX** at $y=6$.



PRACTICE

Given the **FOCUS** to be $(2,5)$ and the **DIRECTRIX** to be $y=3$, find the equation of the parabola.



Given the quadratic equation $y=-2x^2+12x-16$ find

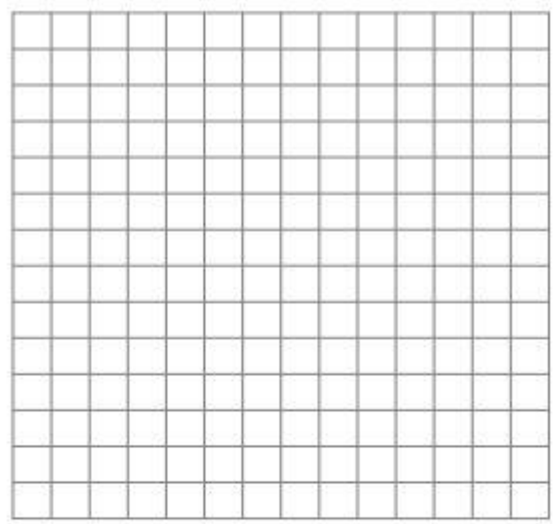
(a) the equation in **STANDARD FORM**

(b) the coordinates of the **VERTEX**

(c) the equation of the **DIRECTRIX**

Write an equation for each parabola described below. Then graph the equation and answer the given questions.

1. Vertex (0, 1) and Focus (0, 4)



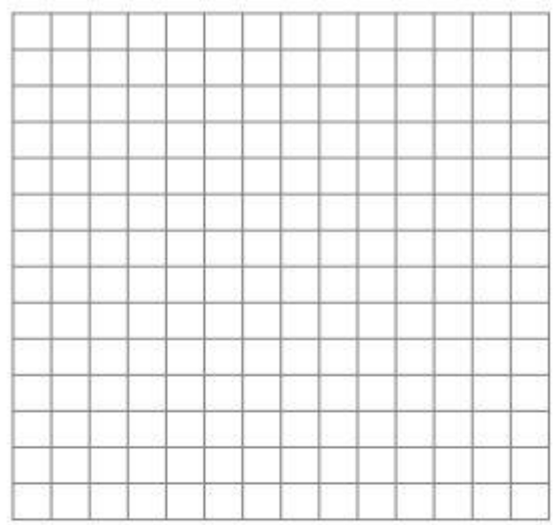
Equation: ________________________

What is the value of p: ________

Equation of the directrix: ________________

Focal Width: __________ Draw it in the graph and Label the two points on the graph.

2. Vertex (1, 8) and Directrix y = 3



Equation: ________________________

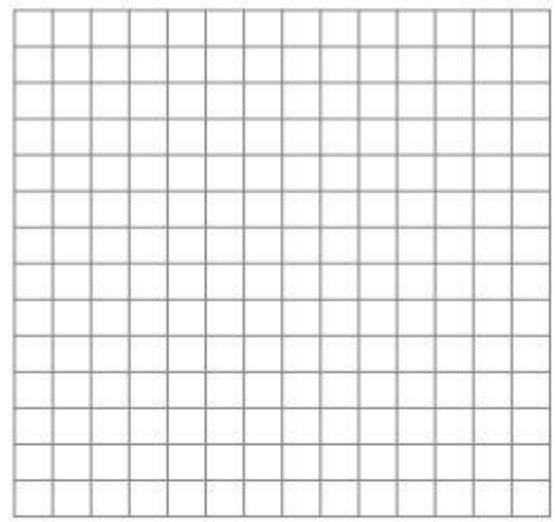
What is the value of p: ________

Equation of the directrix: ________________

Focal Width: __________ Draw it in the graph and Label the two points on the graph.

3. Vertex (9, 6) and Focus (9, 5)

Equation: ________________________



What is the value of p: ________

Equation of the directrix: ________________

Focal Width: __________ Draw it in the graph and Label the two points on the graph.

Module 1 – Polynomial, Rational, and Radical Relationships

Imaginary Numbers

- Imaginary unit "*i* ", defined as $\sqrt{-1}$
- Powers of "i" are cyclical:

  $i^0 = 1$

  $i^1 = i$

  $i^2 = -1$

  $i^3 = -i$

- To simplify higher powers of "i":
  - Divide by 4 and use the remainder as the new power.

    For example:

    $i^{10} = i^2 = -1$

    $i^{47} = i^3 = -i$

    $i^8 = i^0 = 1$

  - Apply two laws of exponents:
    - $x^{ab} = \left(x^a\right)^b$
    - $x^{a+b} = x^a \bullet x^b$

- When multiplying square roots involving negative radicands, **<u>DO NOT</u>** immediately multiply radicands.

  - $\sqrt{a} \bullet \sqrt{b} \neq \sqrt{ab}$ where "a" and "b" are negative.
  - Rewrite the numbers in terms of "i" first.

    $\sqrt{-8} \bullet \sqrt{-2}$

    $\sqrt{-1} \bullet \sqrt{4} \bullet \sqrt{2} \bullet \sqrt{-1} \bullet \sqrt{2}$

    $2i\sqrt{2} \bullet i\sqrt{2}$

    $2i^2\sqrt{4}$

    $2(-1) \bullet 2$

    -4

Module 1 – Polynomial, Rational, and Radical Relationships

Simplify each of the following:

1. i^{12}

2. i^{10}

3. i^{39}

4. i^{72}

5. $i^{5}+i^{12}$

6. $2i^{6}-3i^{2}$

7. $i^{16}-i^{22}$

8. $-i^{50}+i^{51}$

9. $i^{8}-i^{20}$

10. $i^{3}+i\left(2-i\right)$

11. $\sqrt{-9}\bullet\sqrt{-16}$

12. $\sqrt{-32}\bullet\sqrt{-50}$

13. $\sqrt{-48}\bullet\sqrt{-75}$

14. $\sqrt{-72}\bullet\sqrt{-27}$

15. $i^{3}\bullet i^{7}$

16. $\dfrac{\sqrt{-36}}{-\sqrt{4}}$

17. $\sqrt{-25}+2\sqrt{-36}$

18. $3\sqrt{-8}-\sqrt{-2}+5\sqrt{-72}$

19. $2\sqrt{-50}-\sqrt{-32}$

20. $5\sqrt{-4}+\sqrt{-1}-2\sqrt{-9}$

Module 1 – Polynomial, Rational, and Radical Relationships

Simplify.

1. $\sqrt{-108x^7}$

2. $\sqrt{-81x^6}$

3. $\sqrt{-23} \cdot \sqrt{-46}$

4. $(3\boldsymbol{i})(-2\boldsymbol{i})(5\boldsymbol{i})$

5. $\boldsymbol{i}^{11}$

6. $\boldsymbol{i}^{65}$

7. $(7 - 8\boldsymbol{i}) + (-12 - 4\boldsymbol{i})$

8. $(-3 + 5\boldsymbol{i}) + (18 - 7\boldsymbol{i})$

9. $(10 - 4\boldsymbol{i}) - (7 + 3\boldsymbol{i})$

10. $(7 - 6\boldsymbol{i})(2 - 3\boldsymbol{i})$

11. $(3 + 4\boldsymbol{i})(3 - 4\boldsymbol{i})$

12. $\frac{8 - 6i}{3i}$

13. $\frac{3i}{4 + 2i}$

$a+bi$ $i^2 = -1$

Matching Game
COMPLEX NUMBERS

(Designed to match on-line game.)

Name ______________________________

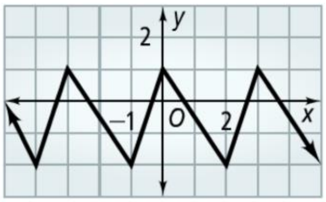
Match the complex number expressions in column I with their equivalent matches in column II. Write the matching choice next to the question number.

I		II	
1.____	$(4 + 3i) - (3 + 4i)$	A	$-7i$
2.____	$(8 - 5i) + (9 + 2i)$	B	18
3.____	$(4 - 3i)(3 + 2i)$	C	20
4.____	$(35i^{16}) \div (5i^5)$	D	$1 - i$
5.____	$(9i^2)(2i^2)$	E	$- 18$
6.____	$(7 - 4i)(2 + 2i)$	F	$18 - i$
7.____	$(4 + 2i)(4 - 2i)$	G	$7i$
8.____	$(6i)(3i)$	H	$22 + 6i$
9.____	$(14i^2) \div (2i)$	I	$- 4$
10.____	$(72i^{45}) \div (18i^{27})$	J	$17 - 3i$

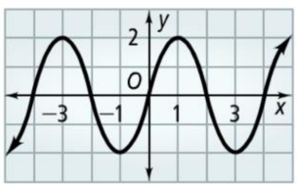
Module 2 – Trigonometric Functions

Exploring Periodic Data

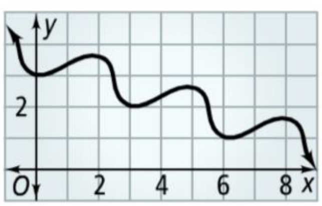
1.



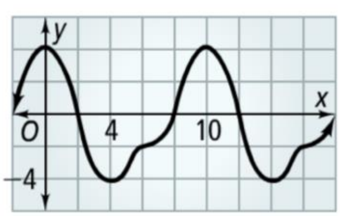
2.



3.

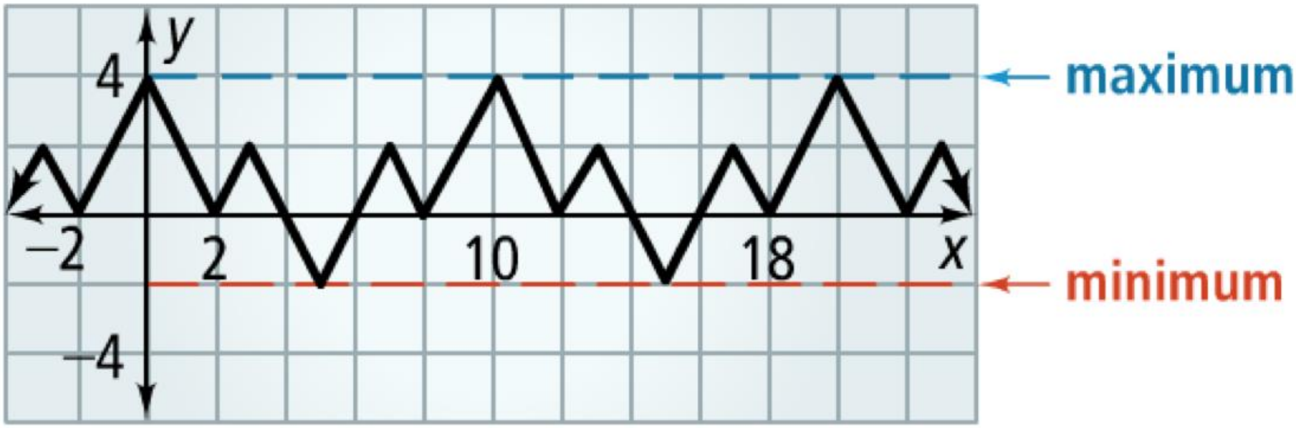


4.



Module 2 – Trigonometric Functions

Periodic Data

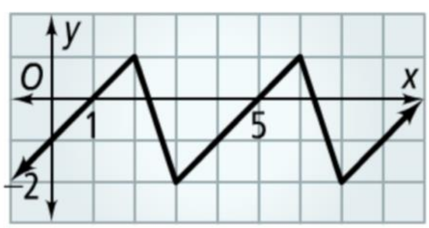


The **MIDLINE** is the horizontal line midway between the maximum and minimum values of a periodic function.

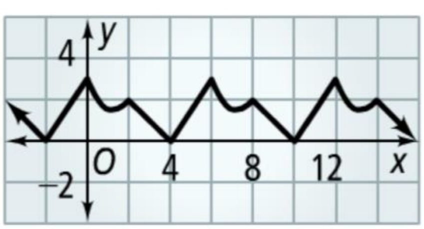
The **AMPLITUDE** is half the distance between the maximum and minimum values of a periodic function.

What is the amplitude and equation of the midline of each period functions shown below?

1.

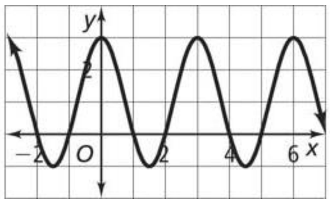


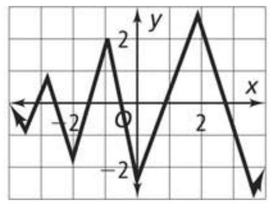
2.

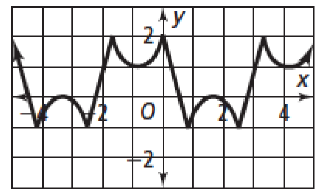


Module 2 – Trigonometric Functions

1. Determine whether each function is or is not periodic. If it is periodic find:
    - ✓ period
    - ✓ amplitude
    - ✓ equation of the midline

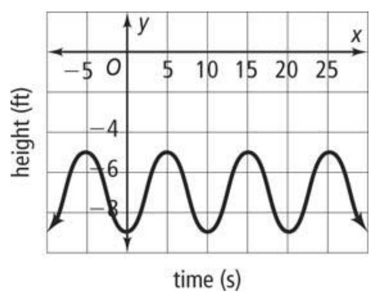






2. The graph on the right shows the height of ocean waves below the deck of a platform.

    (a) What is the period of the graph?
    (b) What is the amplitude of the graph?

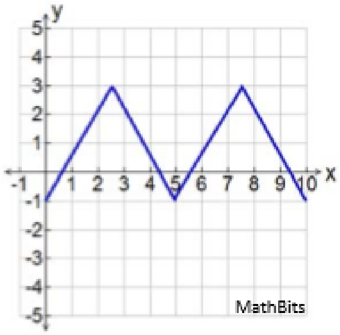


3. Sketch a graph of a periodic function that has a period of 3 and an amplitude of 2.

Module 2 – Trigonometric Functions

<u>Periodic Graphs</u>

1. For the graph below, the investigative interval is 0 to 10.



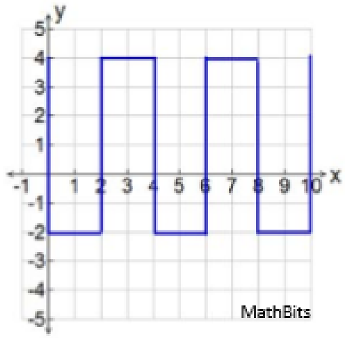
a) Find the amplitude.

b) Find the period.

c) Find the frequency.

d) Find the midline.

2. For the graph below, the investigative interval is 0 to 10.



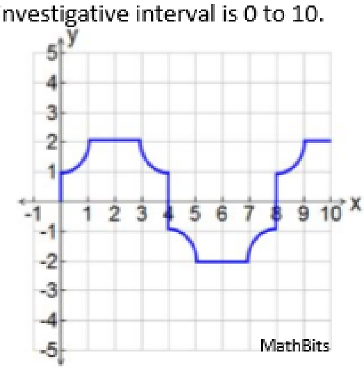
a) Find the amplitude.

b) Find the period.

c) Find the frequency.

d) Find the midline.

3. For the graph below, the investigative interval is 0 to 10.



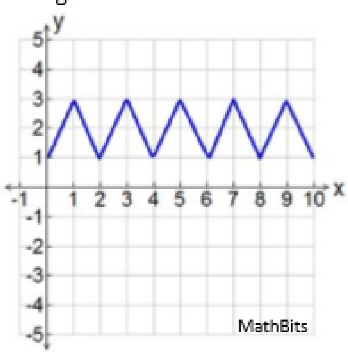
a) Find the amplitude.

b) Find the period.

c) Find the frequency.

d) Find the midline.

4. For the graph below, the investigative interval is 0 to 10.



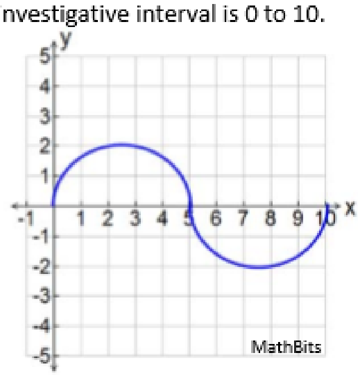
a) Find the amplitude.

b) Find the period.

c) Find the frequency.

d) Find the midline.

5. For the graph below, the investigative interval is 0 to 10.



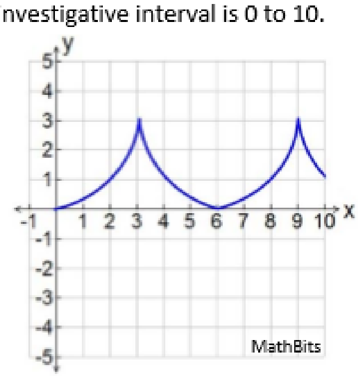
a) Find the amplitude.

b) Find the period.

c) Find the frequency.

d) Find the midline.

6. For the graph below, the investigative interval is 0 to 10.



a) Find the amplitude.

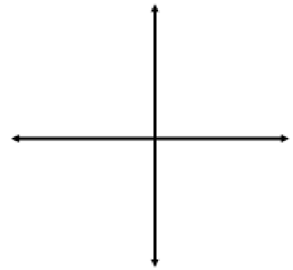
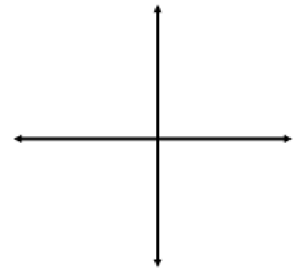
b) Find the period.

c) Find the frequency.

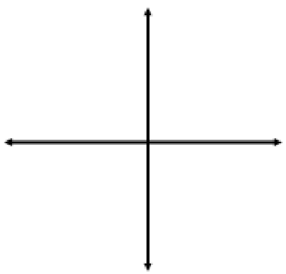
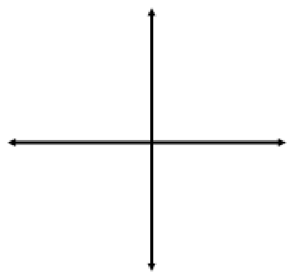
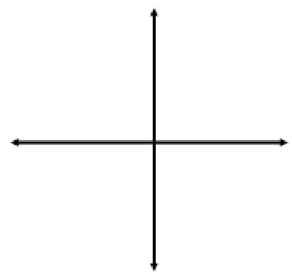
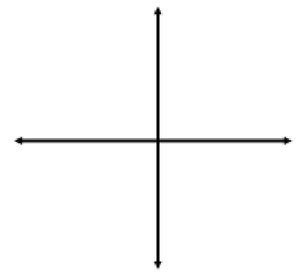
d) Find the midline.

Angles as Rotations

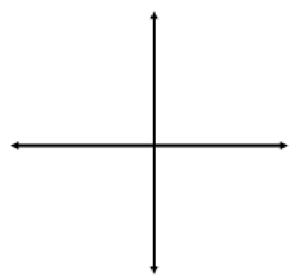
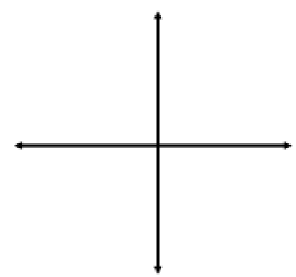
An angle in **STANDARD POSITION** has the **INITIAL SIDE** on the positive x-axis and opens **COUNTER-CLOCKWISE** if it is a positive angle and **CLOCKWISE** if it is a negative angle.

QUADRANTAL ANGLES are angles that terminate on an axis.

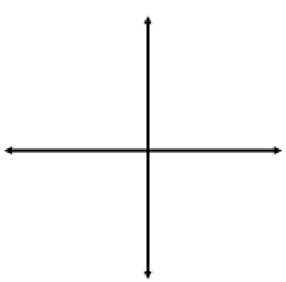
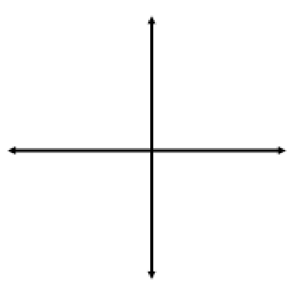
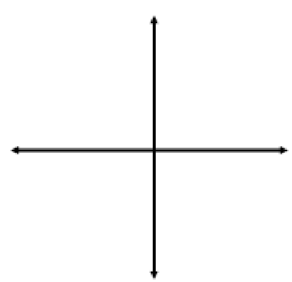
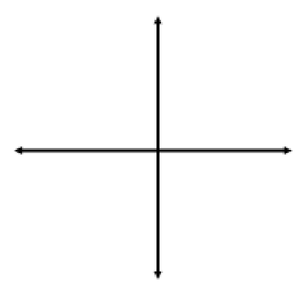





COTERMINAL ANGLES are angles in standard position that have the same terminal side. If two angles are co-terminal the difference of their measures is 360 degrees or a multiple of 360 degrees.

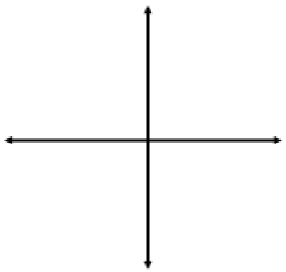
Draw each angle and identify the quadrant where the terminal side lies.

1. −140°
2. −210°
3. −97°
4. −100°

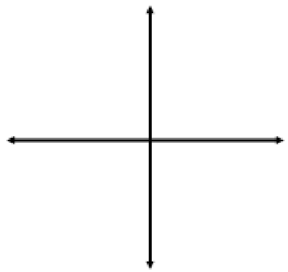





Draw each angle – then find the angle of smallest positive measure that is co-terminal with the given angle.

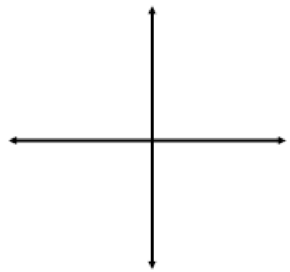
5. - 400°



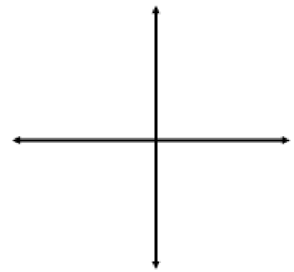
6. - 520°



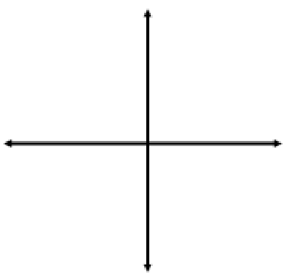
7. - 200°



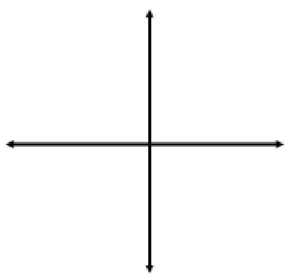
8. - 450°



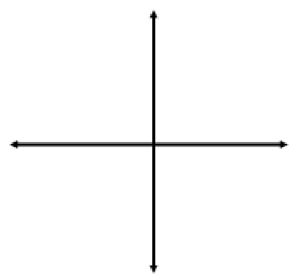
9. 790°



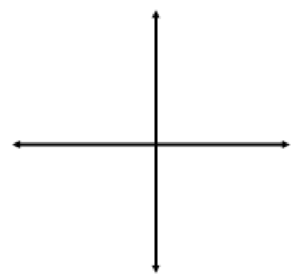
10. - 30°



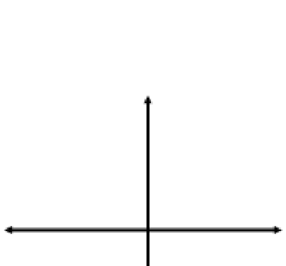
11. 580°



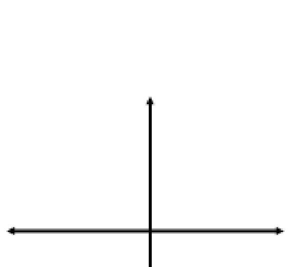
12. - 120°



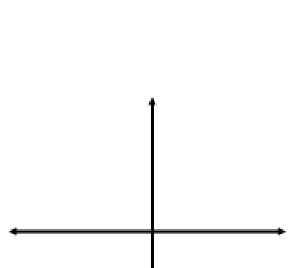
13. - 370°



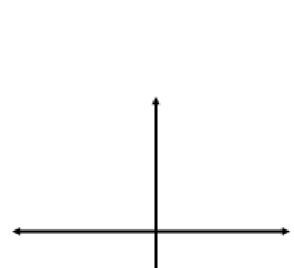
14. 800°



15. - 75°

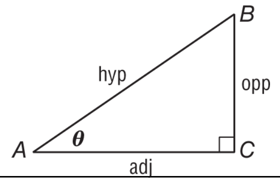


16. - 305°



Module 2 – Trigonometric Functions

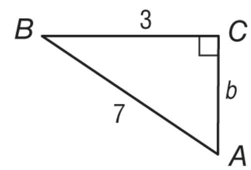
Trigonometric Functions for Acute Angles Trigonometry is the study of relationships among the angles and sides of a right triangle. A **trigonometric function** has a rule given by a **trigonometric ratio**, which is a ratio that compares the side lengths of a right triangle.

Trigonometric Functions in Right Triangles	If ϑ is the measure of an acute angle of a right triangle, *opp* is the measure of the leg opposite ϑ, *adj* is the measure of the leg adjacent to ϑ, and *hyp* is the measure of the hypotenuse, then the following are true.
	$\sin \vartheta = \frac{opp}{hyp}$ $\cos \vartheta = \frac{adj}{hyp}$ $\tan \vartheta = \frac{opp}{adj}$ $\csc \vartheta = \frac{hyp}{opp}$ $\sec \vartheta = \frac{hyp}{adj}$ $\cot \vartheta = \frac{adj}{opp}$

Example: In a right triangle, <*B* is acute and cos *B* = $\frac{3}{7}$. Find the value of tan *B*.

Step 1 Draw a right triangle and label one acute angle *B*. Label the adjacent side 3 and the hypotenuse 7.

Step 2 Use the Pythagorean Theorem to find *b*.



$a^2 + b^2 = c^2$ Pythagorean Theorem

$3^2 + b^2 = 7^2$ *a* = 3 and *c* = 7

$9 + b^2 = 49$ Simplify.

$b^2 = 40$ Subtract 9 from each side.

$b = \sqrt{40}$ or $2\sqrt{10}$ Take the positive square root of each side.

Step 3 Find tan *B*.

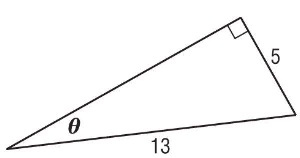
$\tan B = \frac{opp}{adj}$ Tangent function

$\tan B = \frac{2\sqrt{10}}{3}$ Replace *opp* with $2\sqrt{10}$ and *adj* with 3.

Exercises

Find the values of the six trigonometric functions for angle ϑ.

1.



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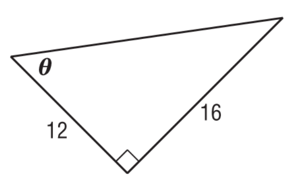

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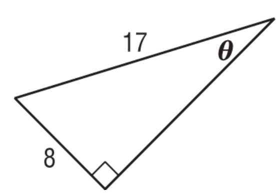


In a right triangle, <*A* and <*B* are acute.

4. If tan *A* = $\frac{7}{12}$, what is cos *A*?

5. If cos *A* = $\frac{1}{2}$, what is tan *A*?

6. If sin *B* = $\frac{3}{8}$, what is tan *B*?

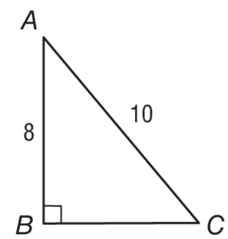
Module 2 – Trigonometric Functions

Use Trigonometric Functions You can use trigonometric functions to find missing side lengths and missing angle measures of right triangles. You can find the measure of the missing angle by using the inverse of sine, cosine, or tangent.

Example: Find the measure of <*C*. Round to the nearest tenth if necessary.

You know the measure of the side opposite <*C* and the measure of the hypotenuse. Use the sine function.

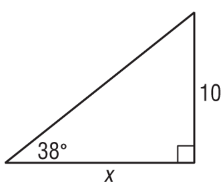
$\sin C = \frac{\text{opp}}{\text{hyp}}$	Sine function
$\sin C = \frac{8}{10}$	Replace *opp* with 8 and *hyp* with 10.
$\sin^{-1}\frac{8}{10} = m\angle C$	Inverse sine
$53.1° \approx m\angle C$	Use a calculator.



Exercises

Use a trigonometric function to find each value of *x*. Round to the nearest tenth if necessary.

1.



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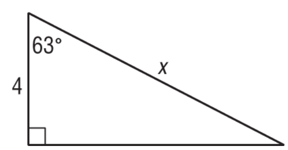

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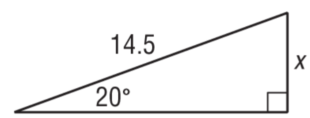

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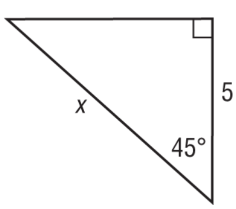

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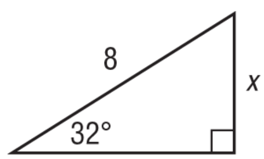

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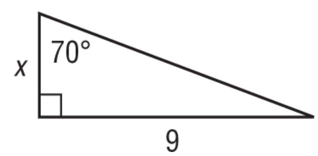
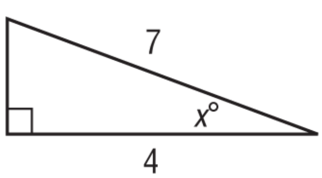


Find *x*. Round to the nearest tenth if necessary.

7.



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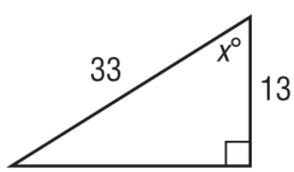

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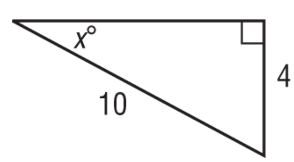
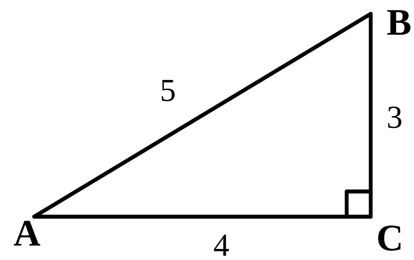


1. Use $\triangle ABC$ to find the following values:



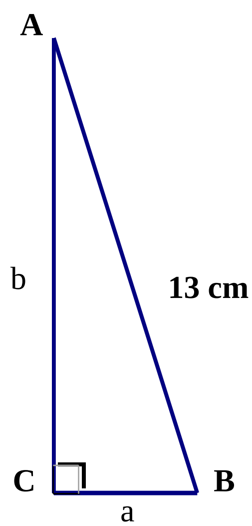
a) $\sin A =$ $\csc A =$

b) $\cos A =$ $\sec A =$

c) $\tan A =$ $\cot A =$

d) Find the measure of $\angle A$ to the nearest degree.

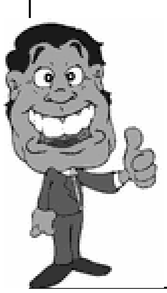
2.



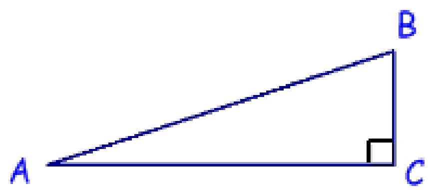
If $\angle A = 19°$, find

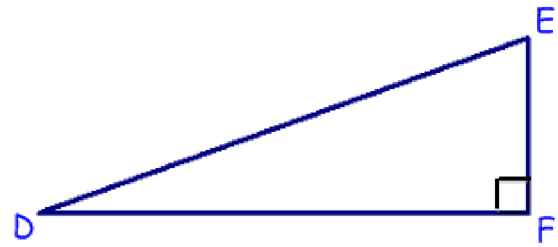
a) $m\angle B$

b) the length of a and b to the nearest tenth of a centimeter.



3. In the diagram below, $\triangle ABC \sim \triangle DEF$.
   If $AB = 17$, $BC = 8$, $AC = 15$, and $DE = 51$, find the following values:





a) $m\angle A$ to the nearest degree

b) $m\angle B$ to the nearest degree

c) $DF =$

d) $EF =$

c) $\sin E =$

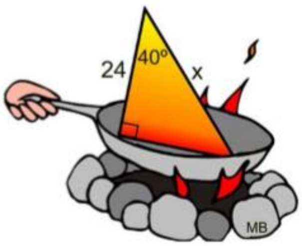
d) $\cos D =$

e) $\sec A =$

f) $\csc E =$

Module 2 – Trigonometric Functions

Directions: Grab your calculator. Solve the following problems rounding your answers to the nearest integer or nearest degree. Please show your work.



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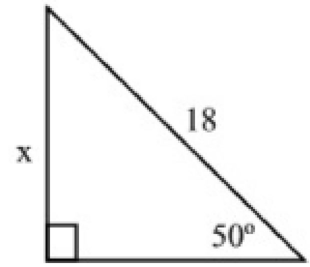

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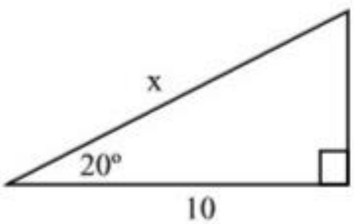

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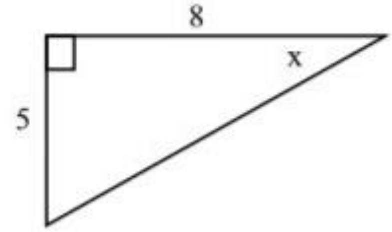

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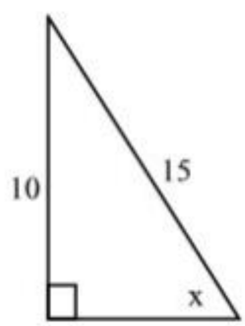

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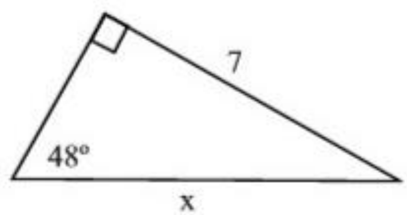

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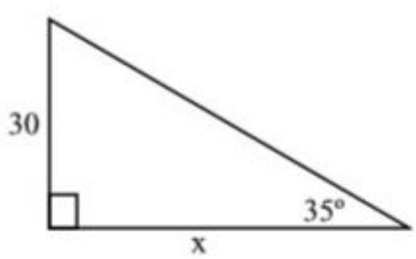

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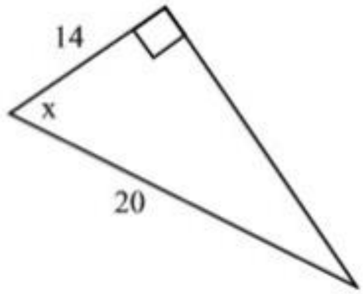
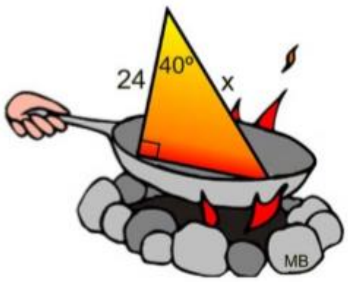
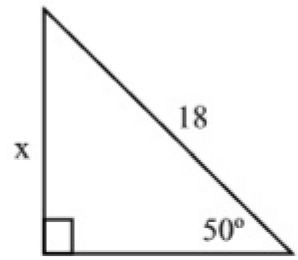


Module 2 – Trigonometric Functions

<u>Directions:</u> Grab your calculator. Solve each problem using ONLY the **reciprocal trigonometric functions** of cosecant, secant, or cotangent. Round your answers to the nearest integer or nearest degree. Please show your work.



1.



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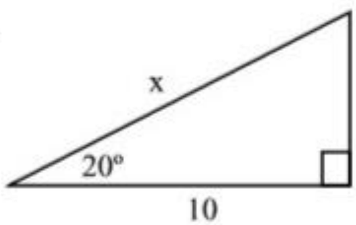

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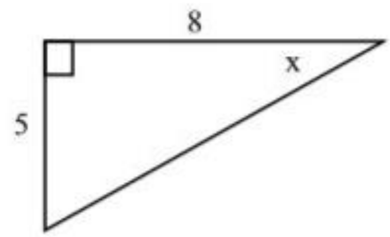

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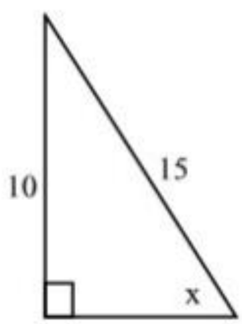

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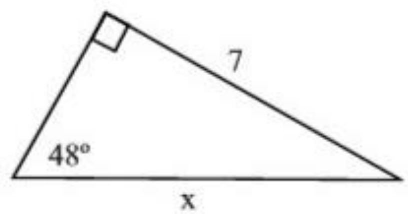

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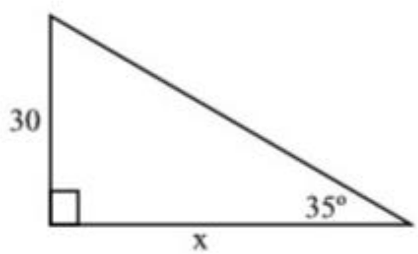

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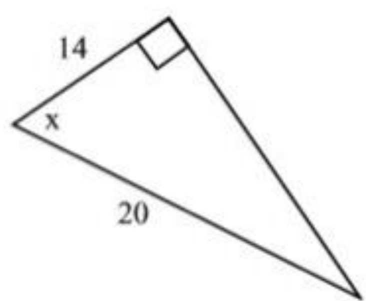
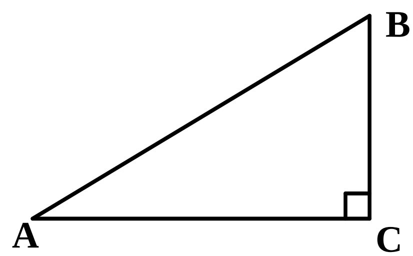
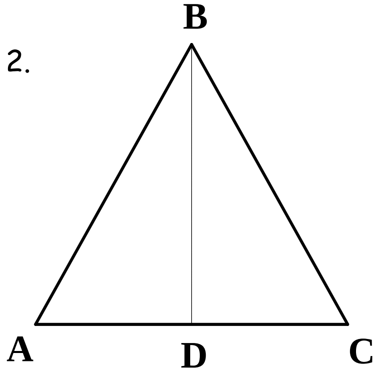


1. In $\triangle ABC$, $m\angle A = 30$. If $AB = 8$, solve the triangle. Explain your reasoning; give *exact values* for all measures.



2. 

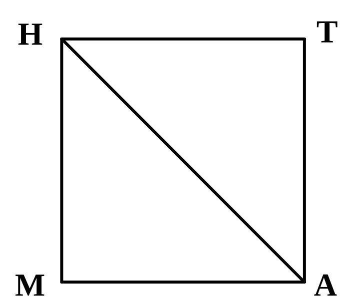
In the diagram at left, $\triangle ABC$ is equilateral; $\overline{BD}$ is an altitude. If $BC = 22$, find the *exact values* of the following:

a) $m\angle C$ b) $m\angle CBD$

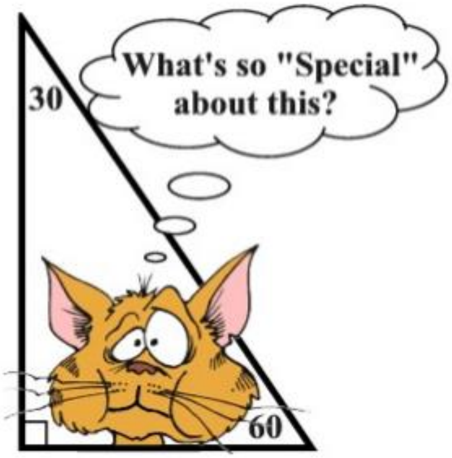
c) $m\angle BDC$ d) DC

e) BD

3. In the diagram at right, $MATH$ is a square with diagonal $\overline{HA}$. If $HA = 20$, what is the exact value of a side of square $MATH$?



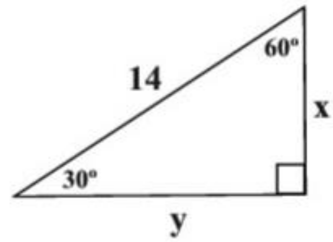
Find $m\angle MHA$. Explain.



SPECIAL RIGHT TRIANGLES

Find the unknown lengths.

1.



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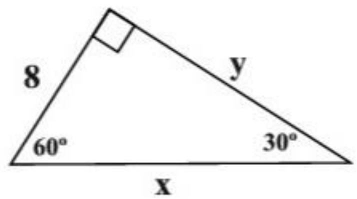

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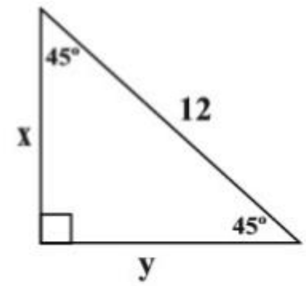

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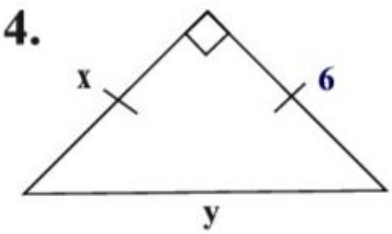

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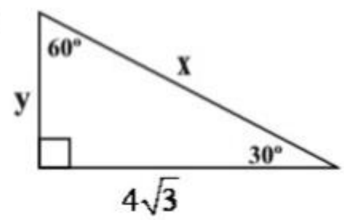

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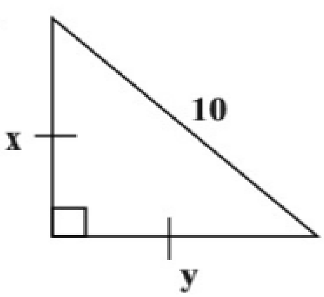

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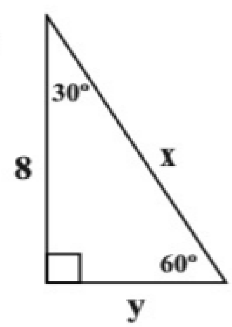
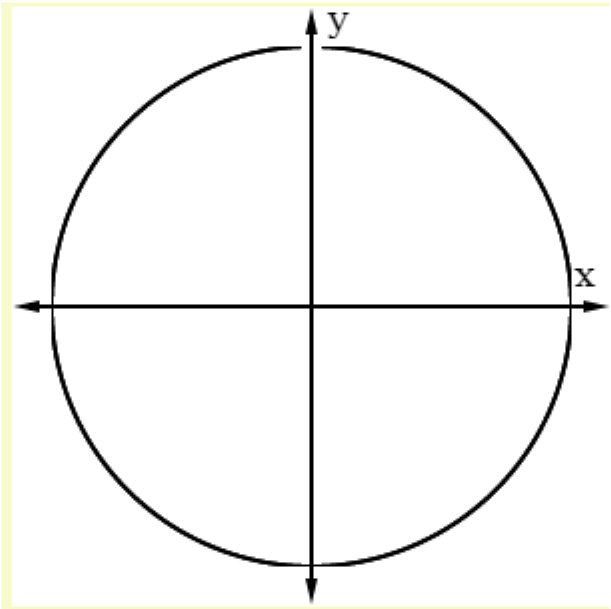
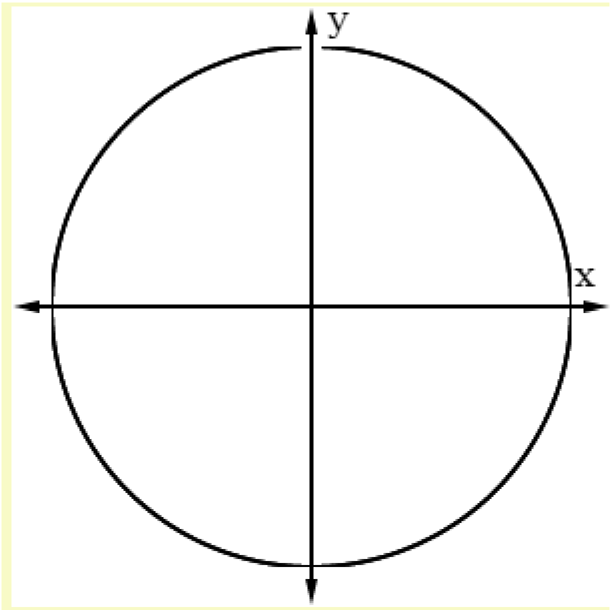


Unit Circle

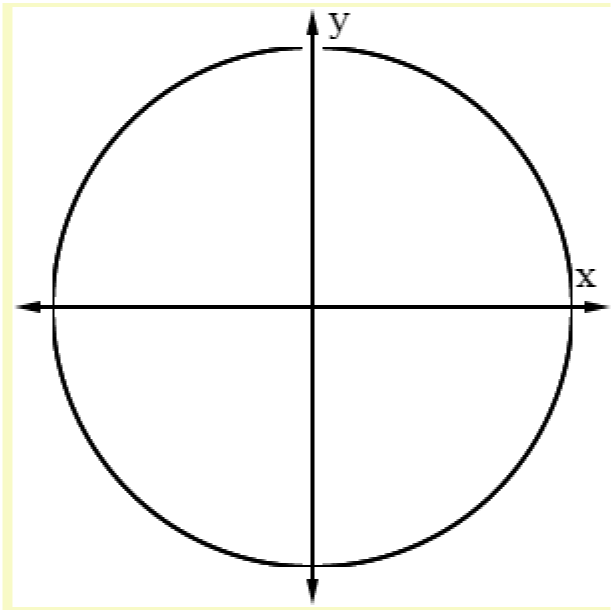
Quadrant I



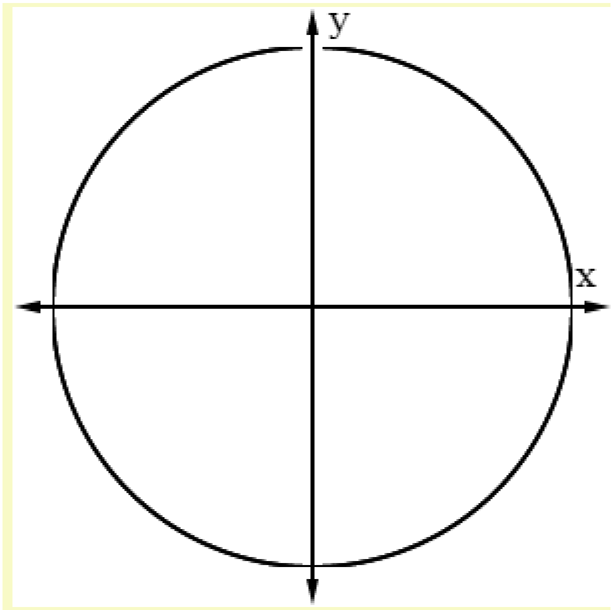
Quadrant II

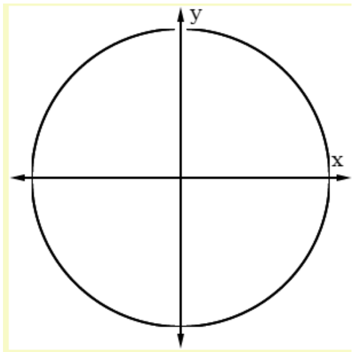


Quadrant III



Quadrant IV

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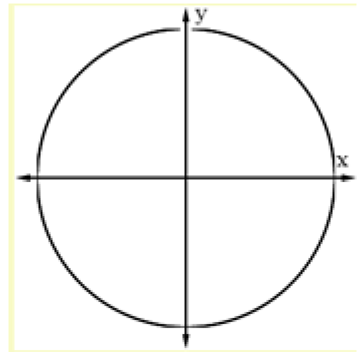
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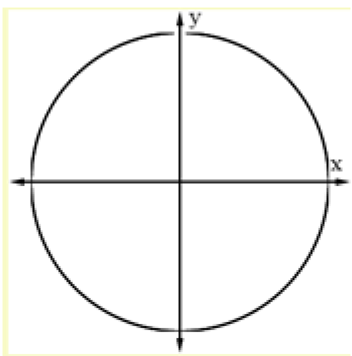
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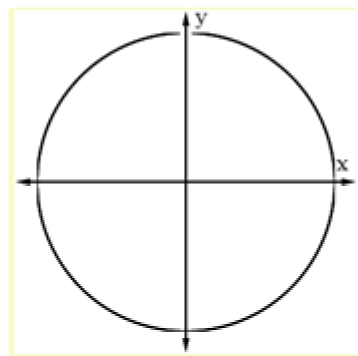
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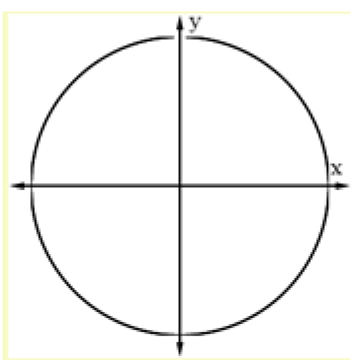
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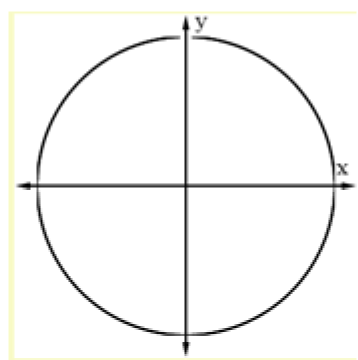
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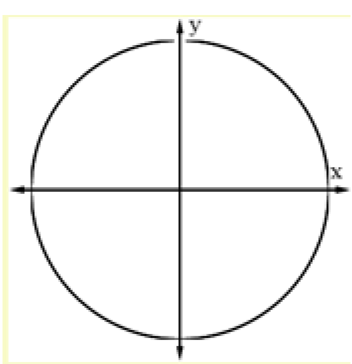
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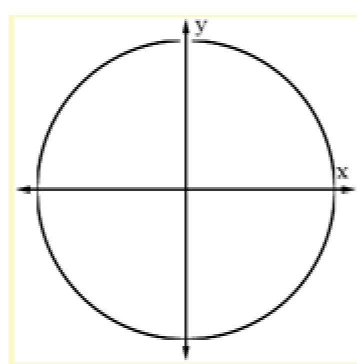
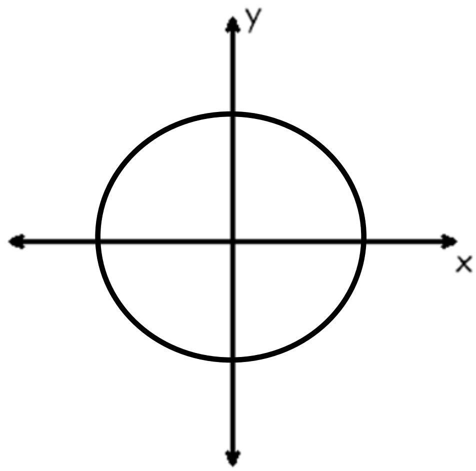

y
x

Module 2 – Trigonometric Functions

1. Draw a reference triangle in each quadrant with a reference angle of 45. Label all sides with their respective values.



2. Find the angles associated with the reference triangle in each quadrant.

I:

II:

III:

IV:

3. Find the exact values of the trigonometric functions for each angle:

I	II	III	IV
Sin 45 =	Sin =	Sin =	Sin =
Cos 45 =	Cos =	Cos =	Cos =
Tan 45 =	Tan =	Tan =	Tan =

4. Explain *in words* the relationship between the trig functions of angles with the same reference angle.

5. Write each function as a function of a positive acute angle. Give the exact value of the function.

(a) tan (240) (b) sin (390) (c) sin (330)

6. In which quadrant would <Θ terminate if:

(a) sin Θ > 0 and cos Θ < 0

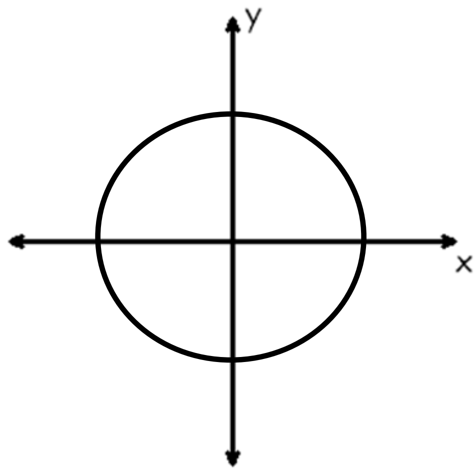
(b) sin Θ, cos Θ, and tan Θ are all positive.

(c) tan Θ > 0 and sin Θ < 0

(d) cos Θ > 0 and sin Θ < 0

Module 2 – Trigonometric Functions

1. Draw a reference triangle in each quadrant with a reference angle of 30. Label all sides with their respective values.



2. Find the angles associated with the reference triangle in each quadrant.

I:

II:

III:

IV:

3. Find the exact values of the trigonometric functions for each angle:

I	II	III	IV
Sin 30 =	Sin =	Sin =	Sin =
Cos 30 =	Cos =	Cos =	Cos =
Tan 30 =	Tan =	Tan =	Tan =

4. Explain *in words* the relationship between the trig functions of angles with the same reference angle.

5. Write each function as a function of a positive acute angle. Give the exact value of the function.

(a) tan (210) (b) sin (390) (c) sin (300)

6. In which quadrant would <Θ terminate if:

(a) sin Θ > 0 and cos Θ < 0

(b) sin Θ, cos Θ, and tan Θ are all positive.

(c) tan Θ > 0 and sin Θ < 0

(d) cos Θ > 0 and sin Θ < 0

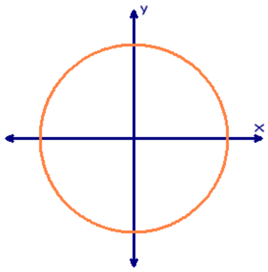
Module 2 – Trigonometric Functions

For each of the following examples:

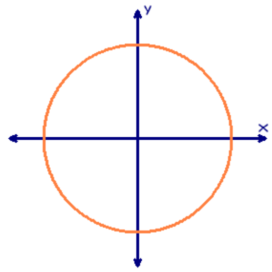
(a) express the given function as a function of a positive acute angle

(b) find the exact function value

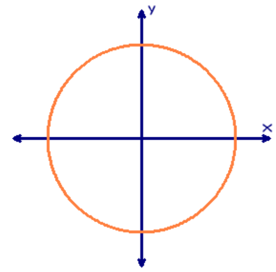
1. Cos 120°



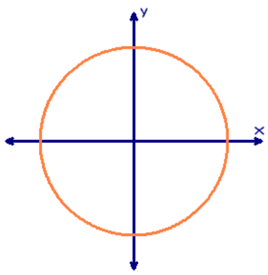
2. Sin 585°



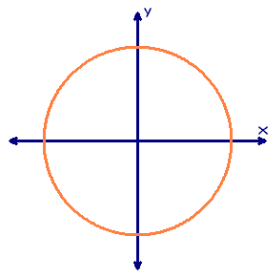
3. Tan 300°



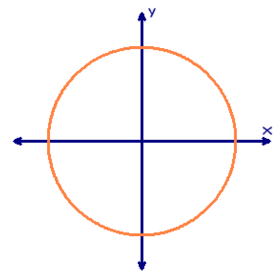
4. Sin 150°



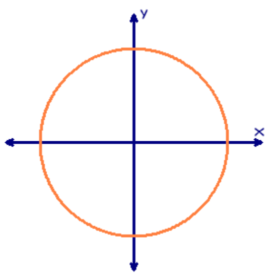
5. Sec 300°



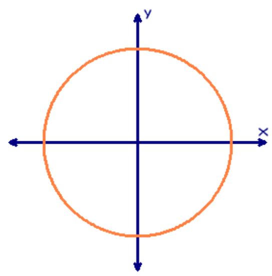
6. Tan 225°



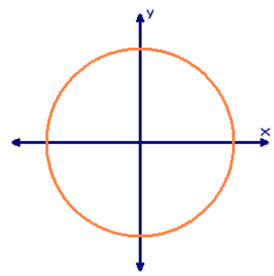
7. Cos (-60)°



8. Cos 135°



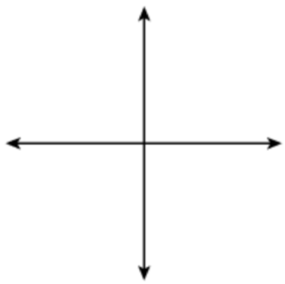
9. Cot 315°



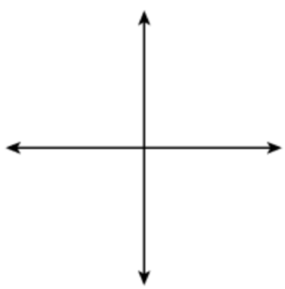
Module 2 – Trigonometric Functions

Let O be the origin and P be a point on the terminal side of ϴ an angle in standard position. Find the measure of the six trigonometric functions and the measure of ϴ to the nearest degree.

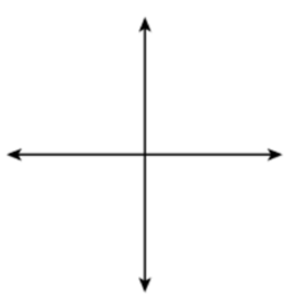
1. P (2, -5)



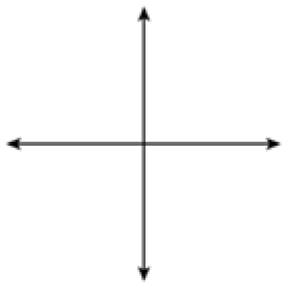
2. P (3, 4)



3. P (-6, -6)

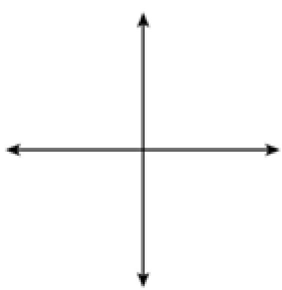


4. P (-2, 3)

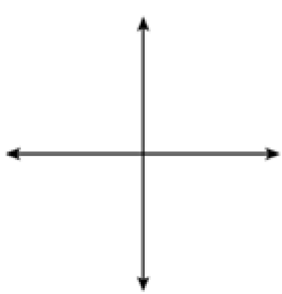


Find the values of the six trigonometric functions of Θ if given the following information.

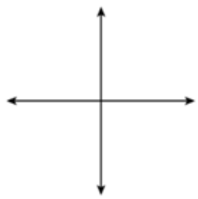
5. $\sin\theta = \frac{-3}{7}$ *and in Quadrant IV*



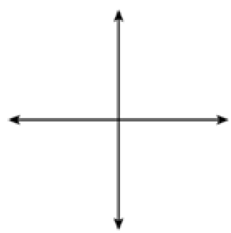
6. $\cot\theta = \frac{2}{5}$ *and* $\cos > 0$



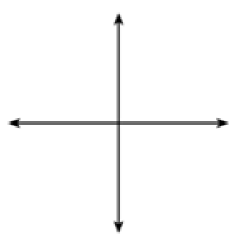
7. $\sin\theta = \frac{-4}{5}, \textit{Quadrant III}$



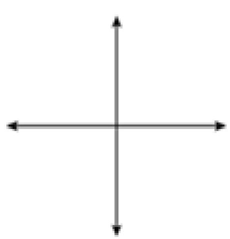
8. $\tan\theta = \frac{-2}{3}, \textit{Quadrant IV}$



9. $\cos\theta = \frac{-8}{17}, \textit{Quadrant III}$



10. $\cot\theta = \frac{-12}{15}, \textit{Quadrant IV}$



Working with Pythagorean Identities

For Questions 1 – 4, use the Pythagorean Identity, $\sin^2\theta + \cos^2\theta = 1$, to support your work. Yes, there are other ways to arrive at the answer, but your task here is to demonstrate how the Identity is involved.

1. If $\cos\theta = \frac{-2}{3}$ and $\tan\theta > 0$, show how to find the value of sine and tangent using a Pythagorean Identity.

$\sin\theta =$	$\tan\theta =$

2. If $\sin\theta = \frac{3}{5}$ and $\tan\theta = \frac{3}{4}$, show how to find the value of cosine using a Pythagorean Identity.

$\cos\theta =$

3. If $\frac{\pi}{2} < \theta < \pi$ and $\sin\theta = \frac{1}{2}$, show how to find the values of cosine and tangent using a Pythagorean Identity.

$\cos\theta =$	$\tan\theta =$

4. If $\pi < \theta < \frac{3\pi}{2}$ and $\cos\theta = \frac{-8}{17}$, show how to find the values of sine and tangent using a Pythagorean Identity.

$\sin\theta =$	$\tan\theta =$

For Questions 5-10, use the Pythagorean Identity, $\sin^2\theta + \cos^2\theta = 1$, to simplify the expressions as directed. More than one solution may be possible.

5. Simplify the expression $(1-\cos^2\theta)(\csc\theta)$ to a single trigonometric function.

6. Simplify $\cos^2\theta + \cos^2\theta\tan^2\theta$.

7. Simplify the complex fraction at the right into a single trigonometric function. $\dfrac{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}{\dfrac{1}{\sin\theta}}$

8. Simplify $\sin^4\theta - \cos^4\theta$ into an expression expressed only in terms of sine.

9. Write the expression $\sec\theta\cos\theta - \cos^2\theta$ as a monomial with a single trigonometric function.

10. Simplify $\dfrac{\sin^2\theta}{1-\sin^2\theta}$.

Module 2 – Trigonometric Functions

Use the Pythagorean Identity, $\sin^2\theta + \cos^2 = 1$, to support your work.

1. If $\cos \theta = -(7/8)$, find the exact value of $\sin \theta$ and $\tan \theta$ if $90° < \theta < 180°$.

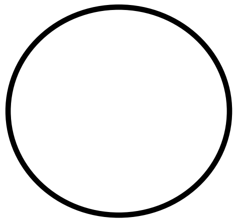
2. If $\sin \theta = -(4/9)$, find the exact value of $\cos \theta$ and $\tan \theta$ if $270° < \theta < 360°$.

3. Simply the expression: $(\sin \theta)(\sec \theta)$

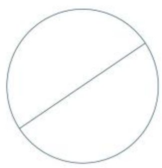
4. Simplify the expression: $\dfrac{1 - \sin^2\theta}{\sin \theta + 1}$

Module 2 – Trigonometric Functions

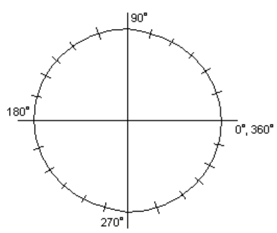
Radians: is a unit measure not a degree value.

1. 

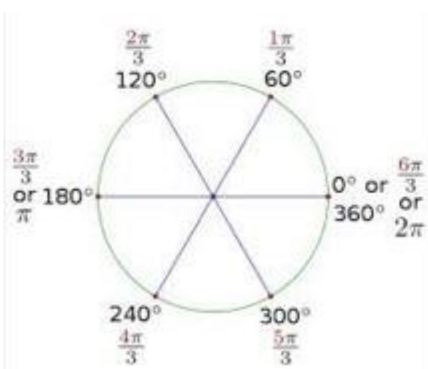
360° = 2π

2. 

180° = π

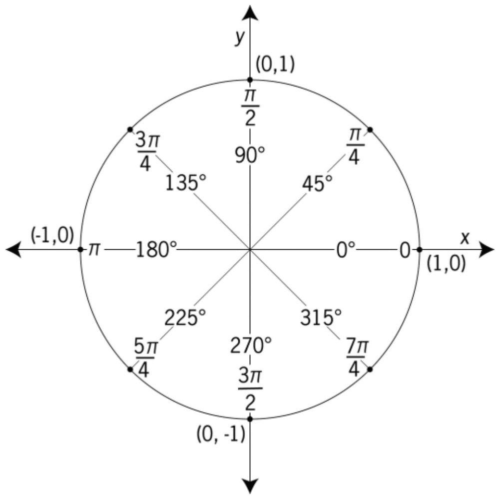
3. 

90° = $\frac{\pi}{2}$

4. 

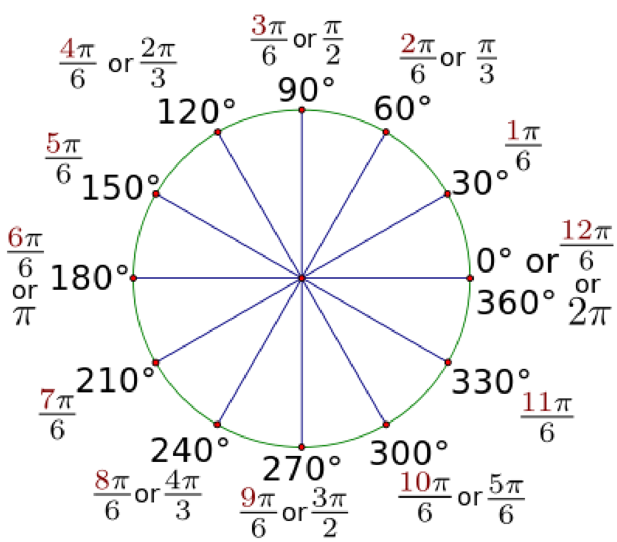
60° = $\frac{\pi}{3}$

5.



$45° = \dfrac{\pi}{4}$

6.



$30° = \dfrac{\pi}{6}$

KeyConcept Convert Between Degrees and Radians

Degrees to Radians	Radians to Degrees
To convert from degrees to radians, multiply the number of degrees by $\frac{\pi \text{ radians}}{180°}$.	To convert from radians to degrees, multiply the number of radians by $\frac{180°}{\pi \text{ radians}}$.

I Convert the following radian measures to degrees:

a) p b) $\frac{p}{2}$ c) $\frac{p}{3}$

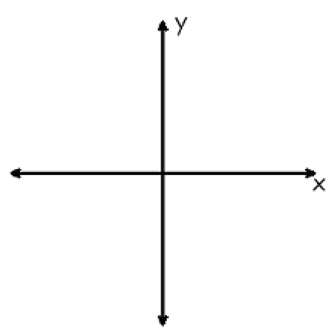
d) $\frac{p}{4}$ e) $\frac{p}{6}$ f) $\frac{2p}{3}$

g) $\frac{4p}{3}$ h) $\frac{5p}{3}$ i) $\frac{3p}{2}$

j) $\frac{3p}{4}$ k) $\frac{5p}{6}$ l) $\frac{7p}{6}$

II Draw a reference triangle for each angle in radian measure. Label the sides of the triangle. Find the exact value of each function.

a) $q = \frac{p}{6}$

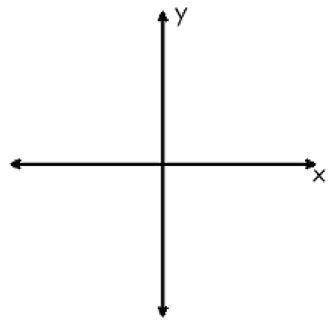


$\sin \frac{p}{6} =$

$\cos \frac{p}{6} =$

$\tan \frac{p}{6} =$

b) $q = \frac{2p}{3}$



$\sin \frac{2p}{3} =$

$\cos \frac{2p}{3} =$

$\tan \frac{2p}{3} =$

III Find the radian measure for each angle:

a) 240° b) 300° c) 405°

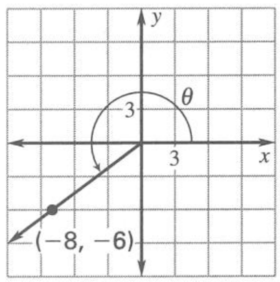
IV Find the exact values for each of the following. Show your method.

a) $\cos \frac{p}{6}$ b) $\sin 60$ c) $\tan \frac{5p}{4}$

d) $\sin \frac{7p}{4}$ e) $\cos \frac{4p}{3}$ f) $\tan \frac{7p}{6}$

Module 2 – Trigonometric Functions

1. Use the given point on the terminal side of an angle θ in standard position.
   Find $\sin\theta$, $\cos\theta$, $\tan\theta$ and the value of θ to the nearest degree.
   Show all work!



2. Sketch the angle, then find its reference angle.

a) -150° b) 315° c) $\frac{7\pi}{4}$

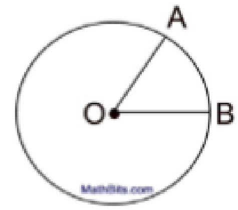
3. Find the exact value of each function.

a) $\sin 300$ b) $\cos\left(\frac{2\pi}{3}\right)$ c) $\tan 135$

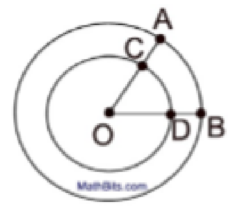
d) $\sin\left(\frac{3\pi}{4}\right)$ e) $\tan 405$ f) $\cos\left(\frac{11\pi}{6}\right)$

Arc Length and Radians

Directions: Read carefully and remember to work in radians.



1. In the circle at the right, the measure of acute $\angle AOB$ is $\frac{7}{4}$ radians and the radius of the circle is 8 centimeters. What is the length of the minor arc $\overset{\frown}{AB}$?



2. Two concentric circles centered at O are shown in the diagram at the right. If $OC = 6$ and $CA = 3$, what is the ratio of $m\overset{\frown}{CD}$ to $m\overset{\frown}{AB}$?

3. In a circle, a central angle of $\frac{1}{3}$ radians subtends an arc of 3 inches. Find the length, in inches, of the radius of the circle.

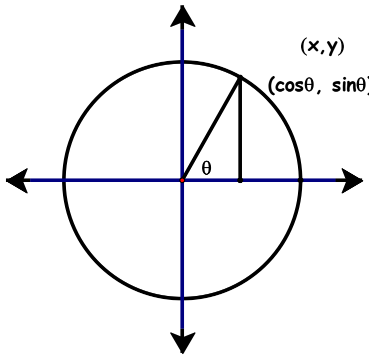
4. Find the measure of the central angle (in radians) subtended by an arc of length 6 centimeters in a circle of radius 4 centimeters.

5. A biking track is built around the circumference of a circular man-made lake. Find the angle (in radians) that represents a biker's positon relative to his/her starting position if he/she rides 6 miles and the lake has a radius of 3 miles?

6. A large circular pizza is divided into eight equal slices. The measure of the outer edge of the crust on one slice of the pizza is 5 inches. What is the diameter of the pizza to the *nearest inch*?

7. The pendulum of a clock swings through an angle of 2.5 radians as its tip travels through an arc of 50 centimeters. Find the length of the pendulum, in centimeters.

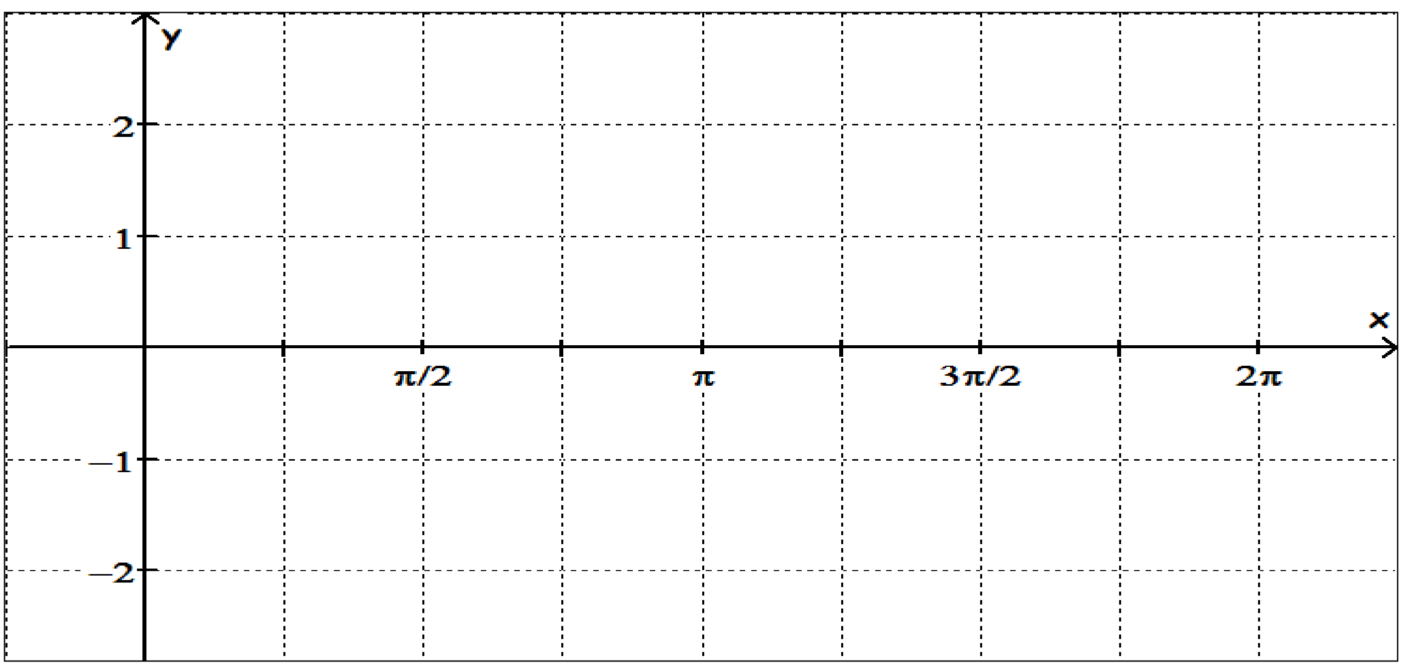
Sine and Cosine Graphs



Recall that in a unit circle the x and y coordinates of points on the circle correspond to the values of $\cos\theta$ and $\sin\theta$, respectively.

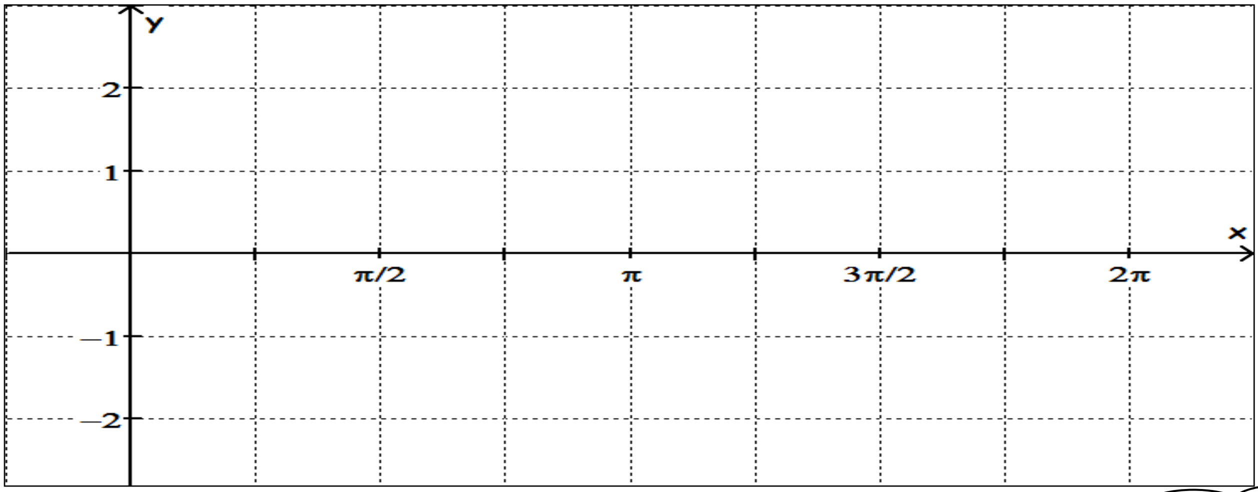
θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin\theta$											
Decimal Values											
$\cos\theta$											
Decimal Values											

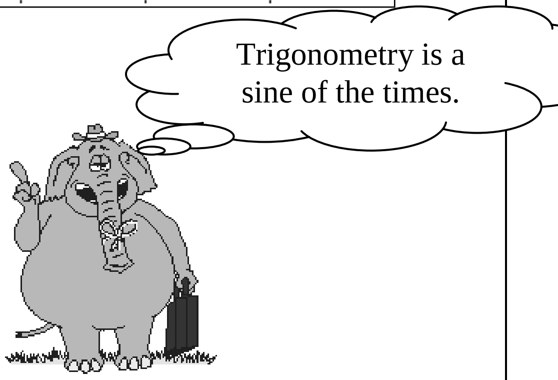
Use the results above to graph the function $y = \sin x$ in the domain $0 \le x \le 2\pi$.



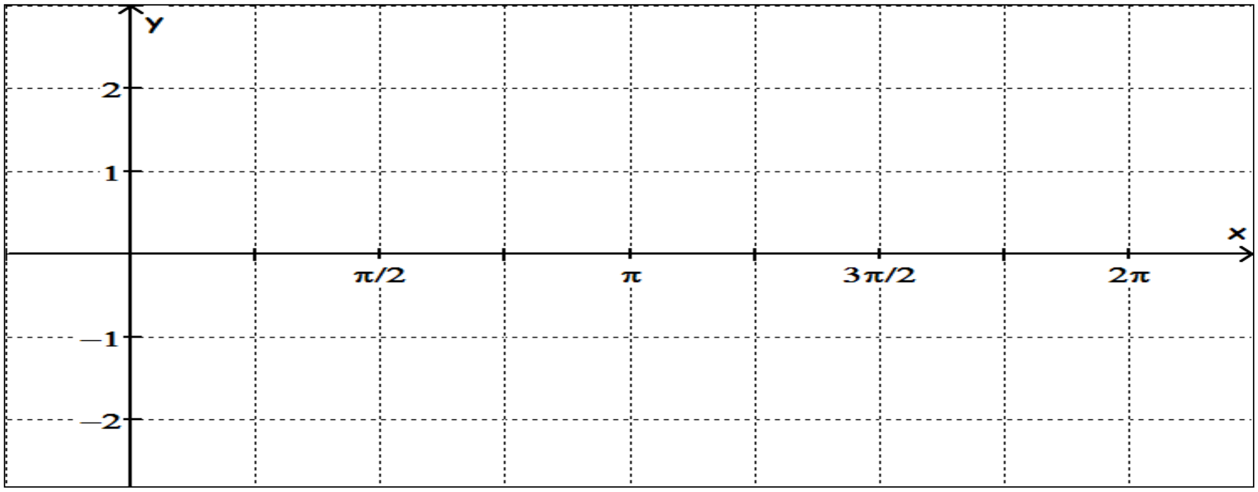
Use your calculator to graph the following equations. Sketch and label below.

a) $y = 2\sin x$

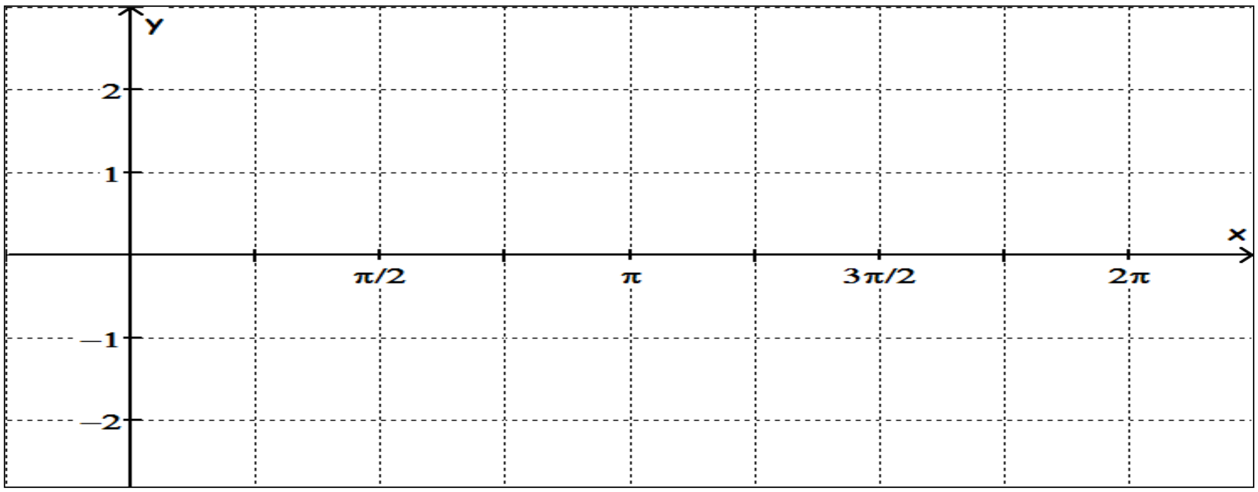




b) $y = -\sin x$



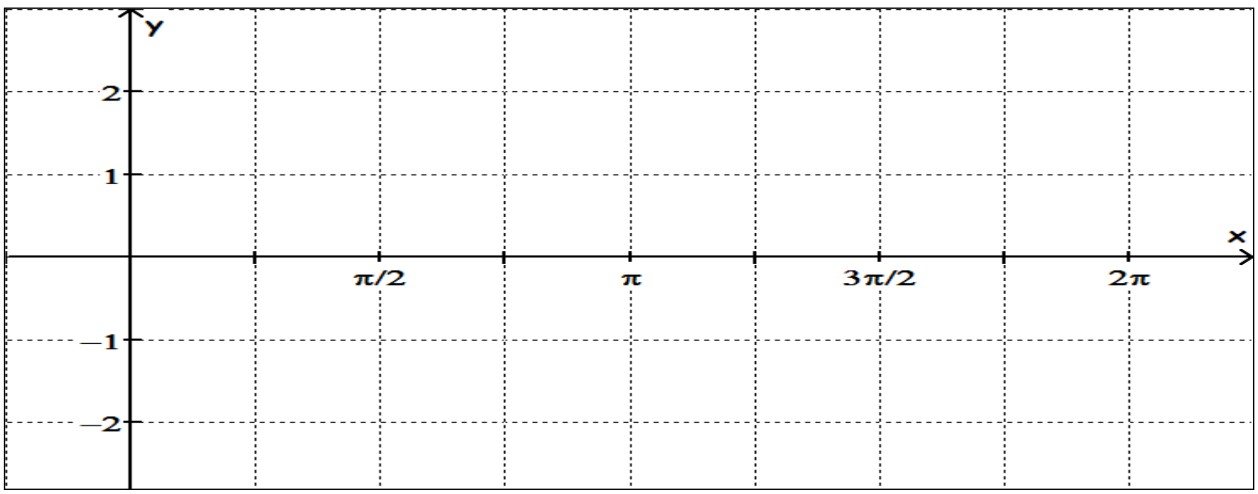
c) $y = \frac{1}{2}\sin x$



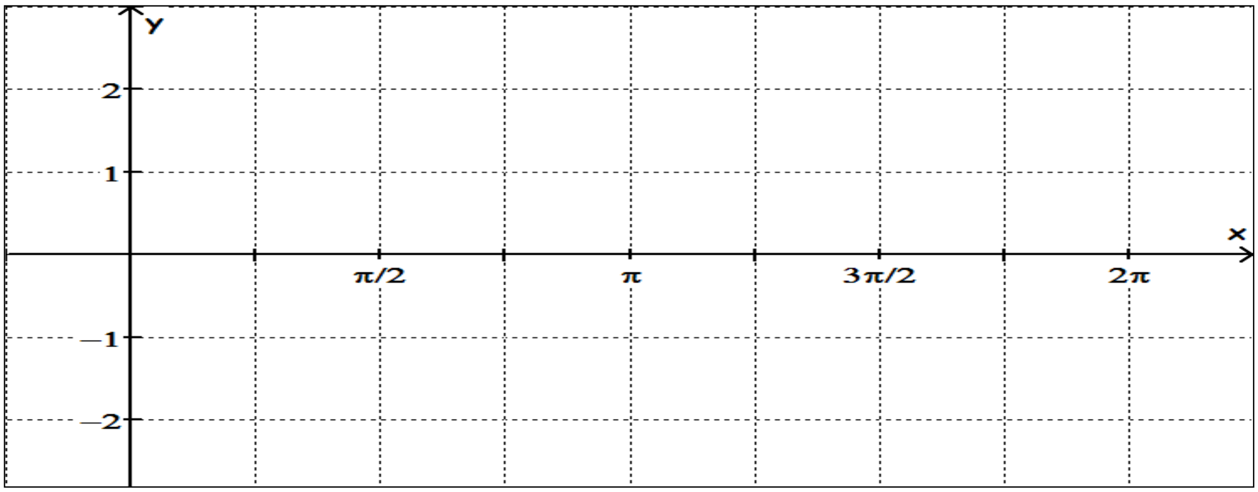
d) In the equation $y = A\sin x$, what effect does the value of *A* have on the graph?

Use your calculator to graph the following; sketch and label below.

e) $y = \sin 2x$

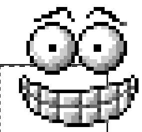


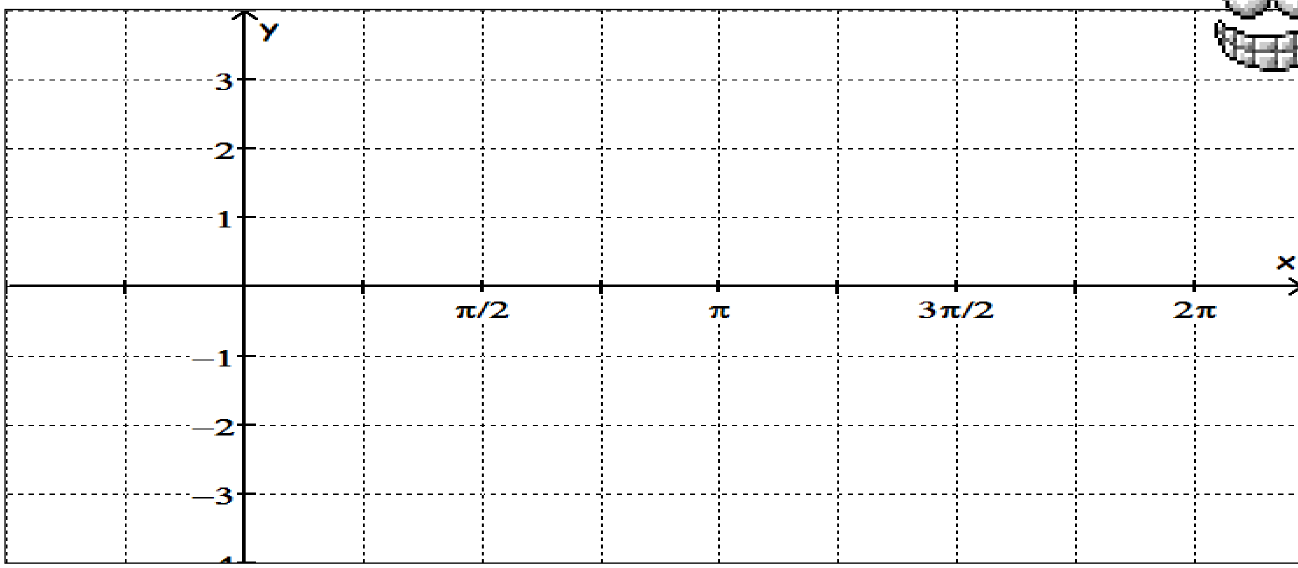
e) $y = \sin\frac{1}{2}x$



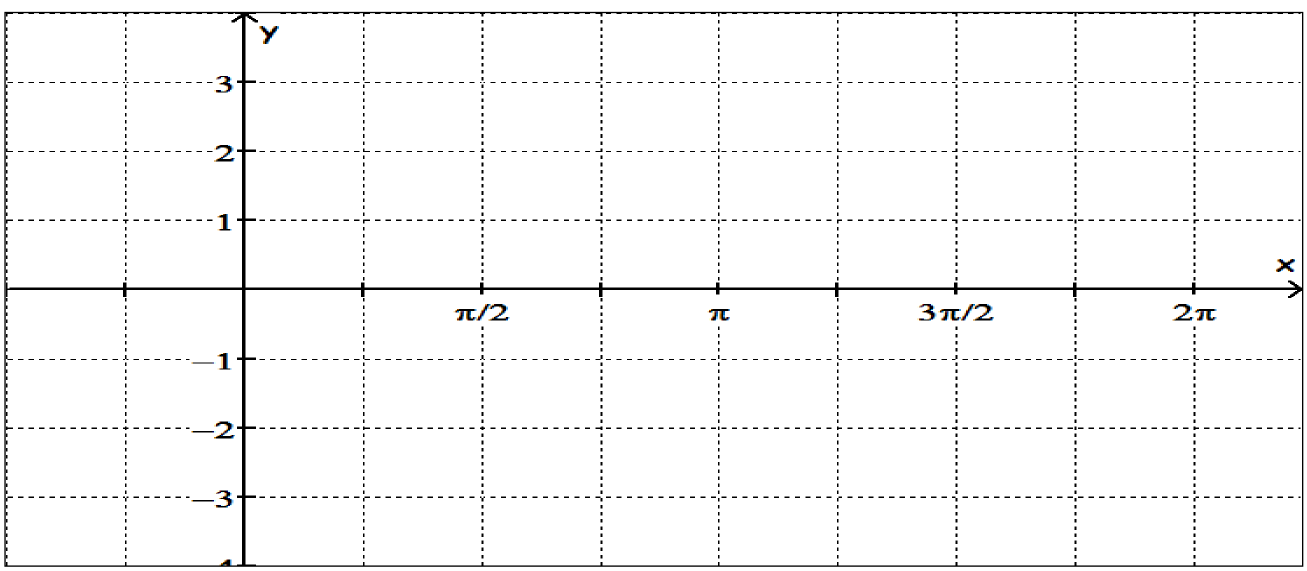
f) In the equation $y = A\sin Bx$, what effect does the value of *B* have on the graph?

1. Graph the "parent" function, $y = \cos x$ in the domain $0 \le x \le 2\pi$.



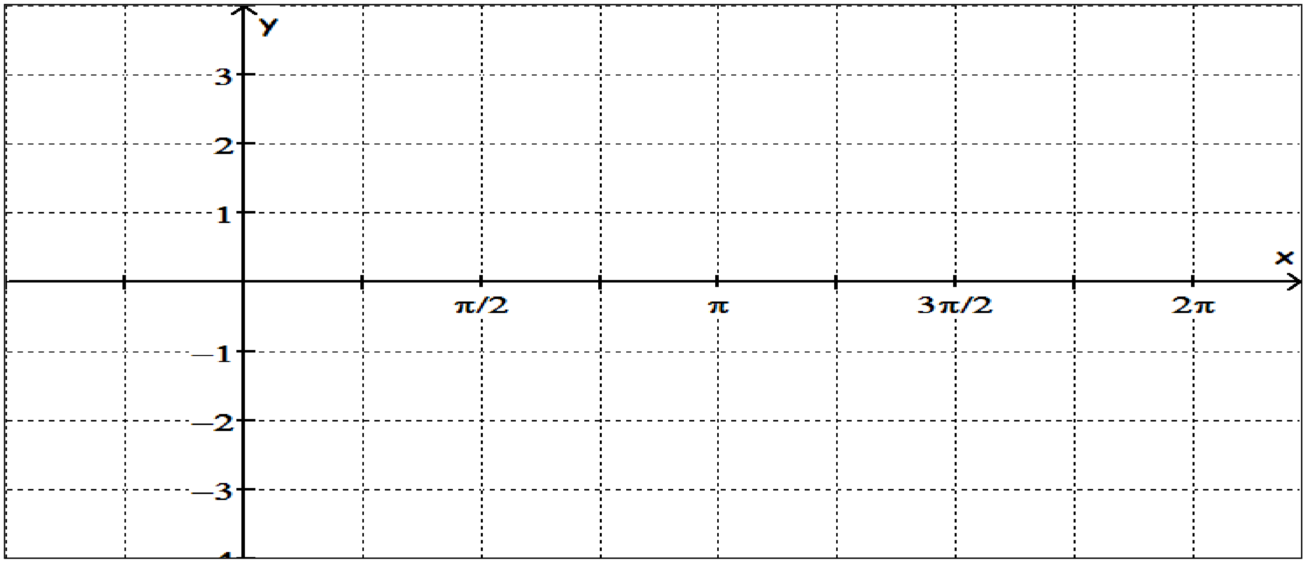


2. Use your calculator to graph $y = \cos x + 2$ on the axes below.



Experiment with other values in your calculator to determine the effect of changing the value of *k* in the equation $y = A\sin Bx + C$. Let $A = 1, \quad B = 1, \quad C = 0$; vary the value of *C*. What do you notice?

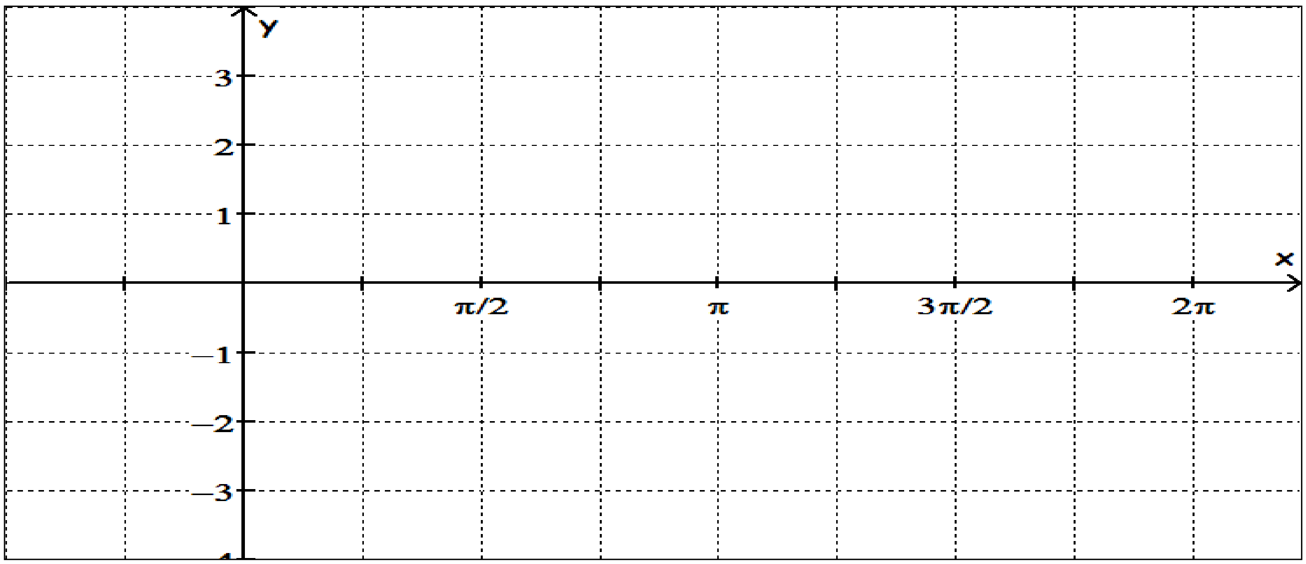
3. Use your calculator to graph $y = \cos 2x$.



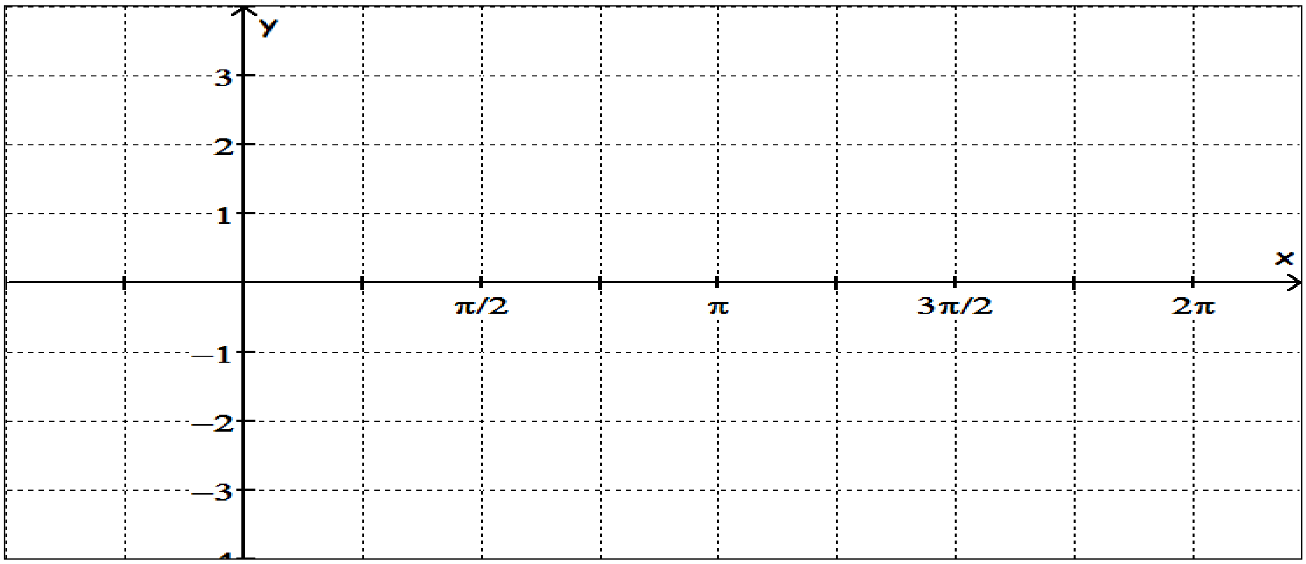
In the equation $y = A\sin Bx + C$, let $A = 1$, $B = 1$, $C = 0$; vary the values of B.
How do changes in B affect the graph of the equation?

4. Sketch the following graphs and state the period of each.

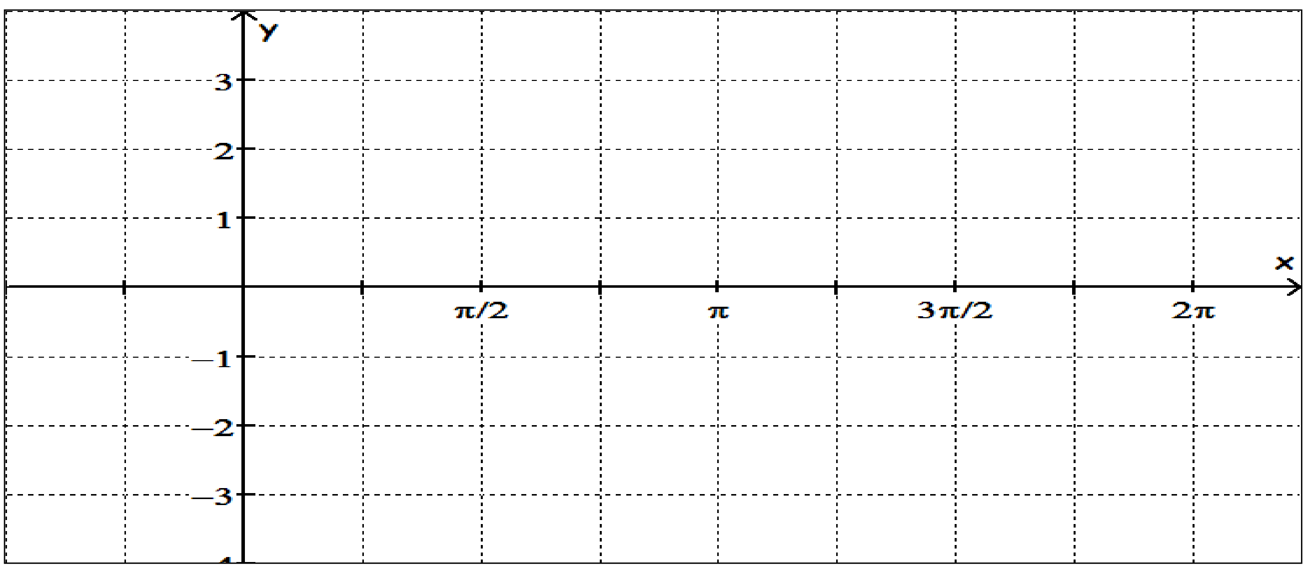
a) $y = 2\cos 2x$



b) $y = -\cos x - 2$

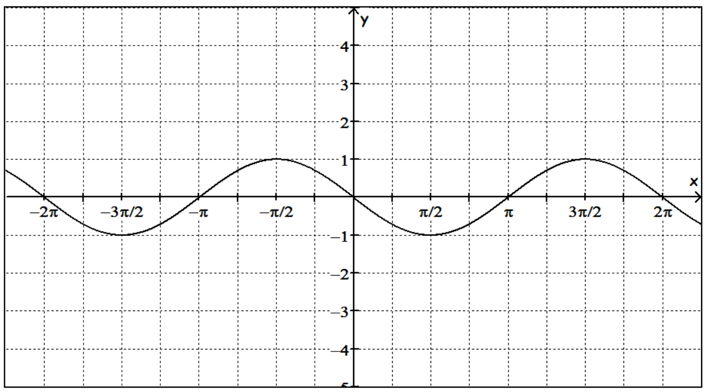


c) $y = \cos \frac{1}{2} x$



$$f(x) = A\sin B(x-h)+k \qquad f(x) = A\cos B(x-h)+k$$

1.



Midline ______________

Amplitude ______________

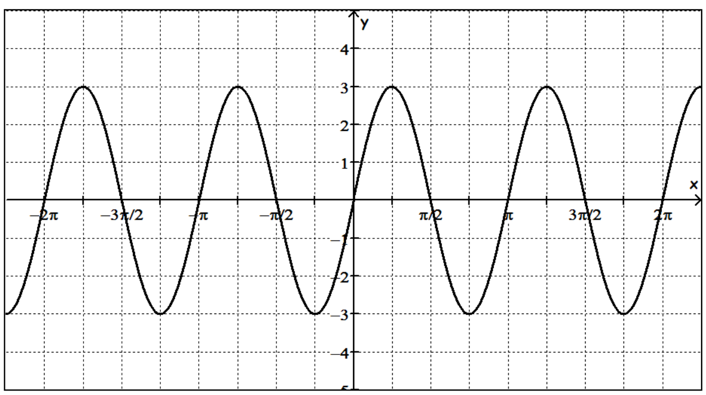
Period ______________

Frequency ______________

Shift (h and/or k) ________

Equation ______________

2.



Midline ______________

Amplitude ______________

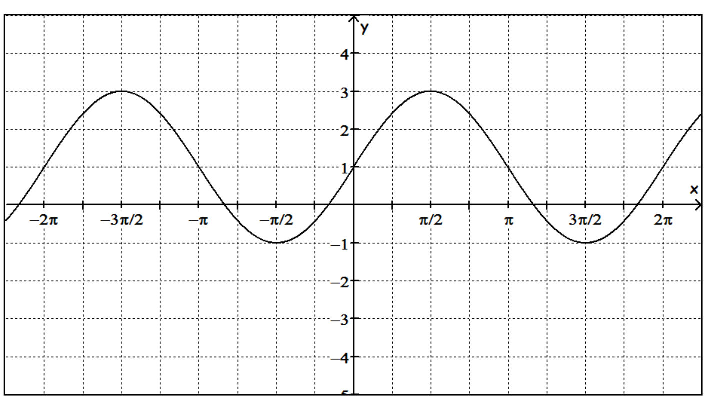
Period ______________

Frequency ______________

Shift (h and/or k) ________

Equation ______________

3.



Midline ______________

Amplitude ______________

Period ______________

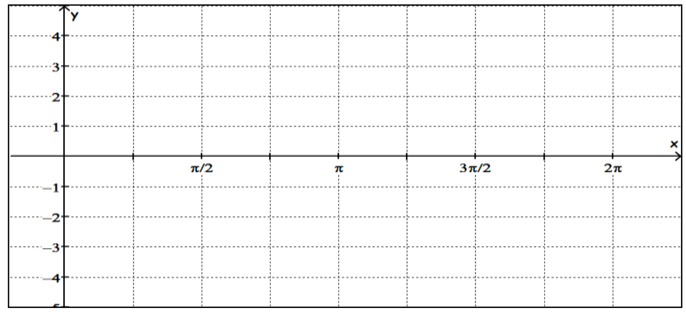
Frequency ______________

Shift (h and/or k) _______

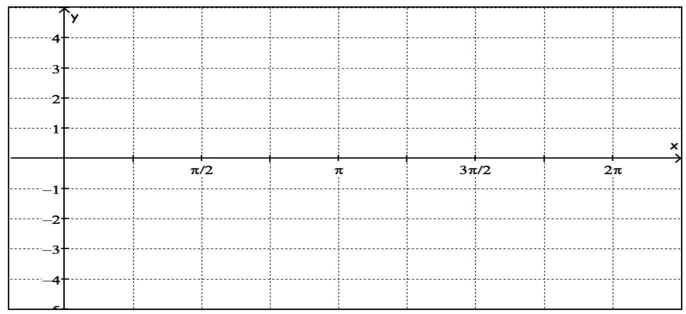
Equation ______________

For each function below identify the amplitude, frequency, midline, period, horizontal and vertical shift.
Sketch one period of the function.

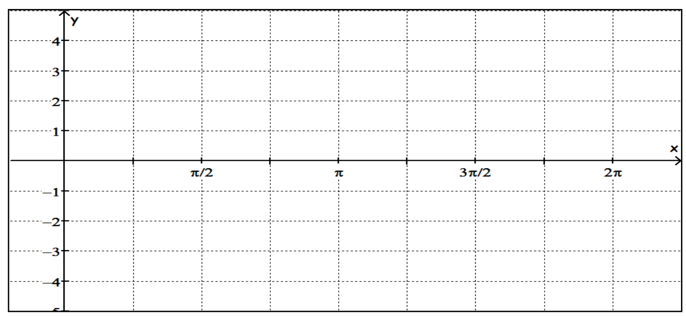
4. $y = -3\sin x$



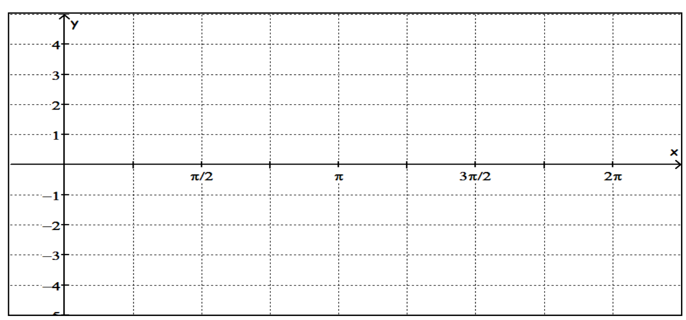
5. $y = 2\cos 2x + 1$



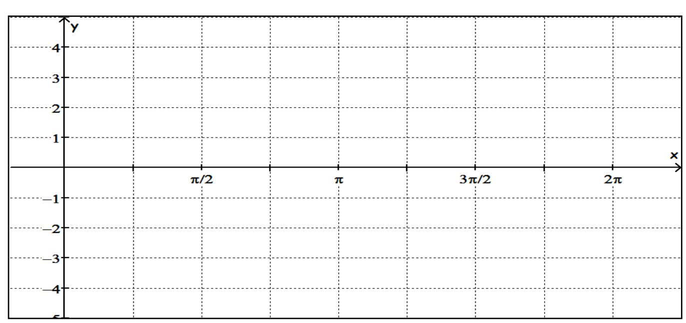
6. $y = \sin x - 2$



7. $y = \cos x - 2$



8. $y = 3\sin\frac{1}{2}x - 2$



1. Define the following terms for periodic functions:

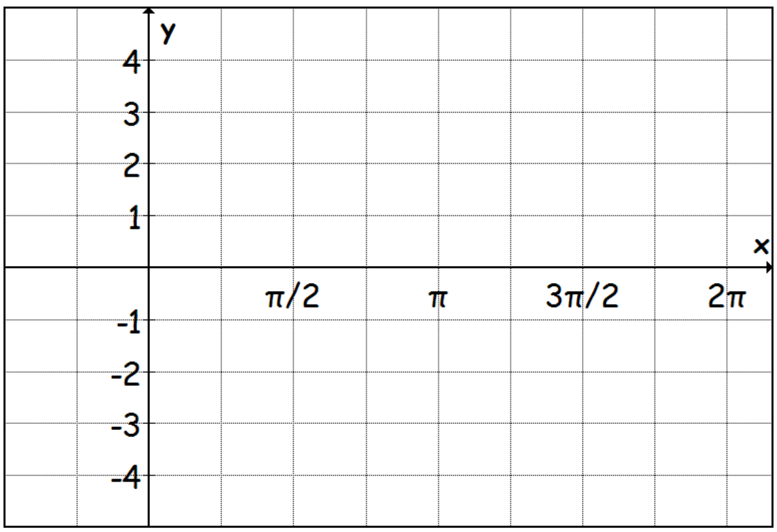
   a) amplitude:

   b) period

   c) frequency

   d) "midline"

2. Graph the "parent " function, $y = \sin x$ in the domain $0 \le x \le 2\pi$.



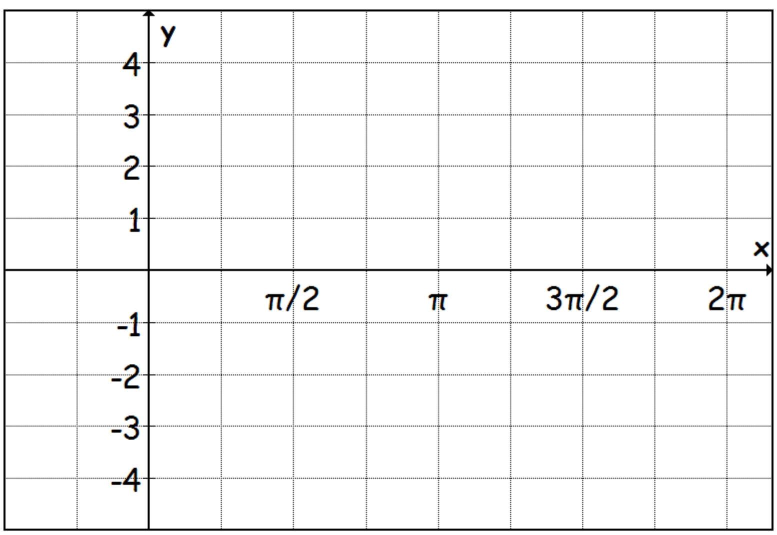
Equation of midline: ________

Amplitude: __________

Frequency: __________

Period: _________

3. Graph the parent function $y = \cos x$ in the domain $0 \le x \le 2\pi$.



Equation of midline: ________

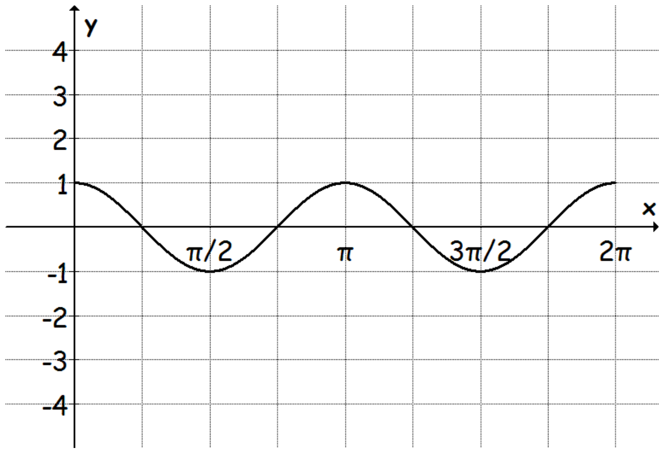
Amplitude: __________

Frequency: __________

Period: _________

4. Write an equation for each graph by determining the values of all parameters.

a)



Equation of midline: __________

Amplitude: _____________

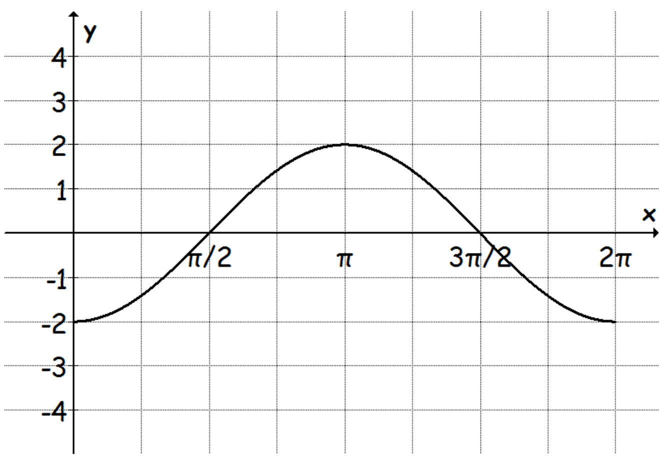
Frequency: _____________

Period: _____________

Equation: _____________

Use the same grid to draw the graph of $y = 3\sin x - 1$.
How is this graph different from the original?

b)



Equation of midline: ___________

Amplitude: _____________

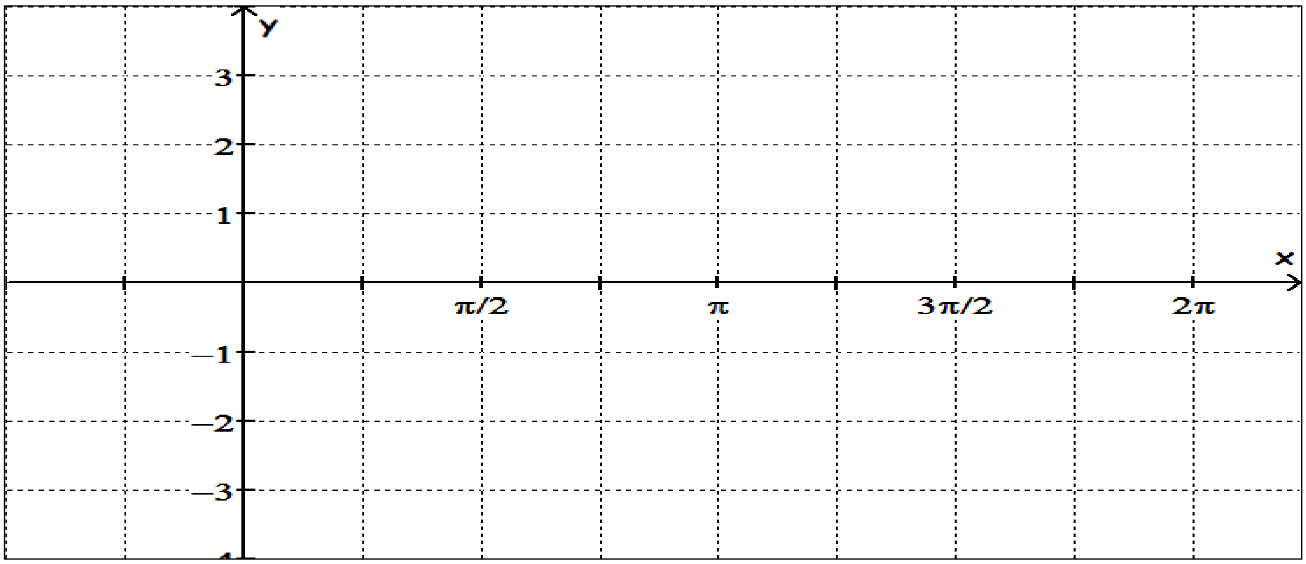
Frequency: _____________

Period: _____________

Equation: _____________

On the same grid draw the graph of $y = 3\cos 2x$.
How is this graph different from the original?

1. Graph $y = \sin x$ and $y = \cos x$ in the domain $0 \le x \le 2\pi$ on the same set of axes below.



a) In what interval(s), $0 \le x \le 2\pi$, are the graphs of $y = \sin x$ and $y = \cos x$ both decreasing?

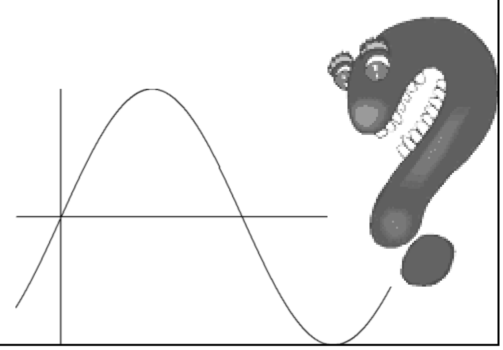
In what interval(s) are they both increasing?

b) How many solutions are there in the domain $0 \le x \le 2\pi$ for the equation $\sin x = \cos x$?

c) At what radian value(s) of x, $0 \le x \le 2\pi$, does $\sin x = \cos x$?

d) At what value(s), $0 \le x \le 2\pi$, does $\sin x - \cos x = 1$?

e) At what value(s), $0 \le x \le 2\pi$, does $\sin x - \cos x = -1$?

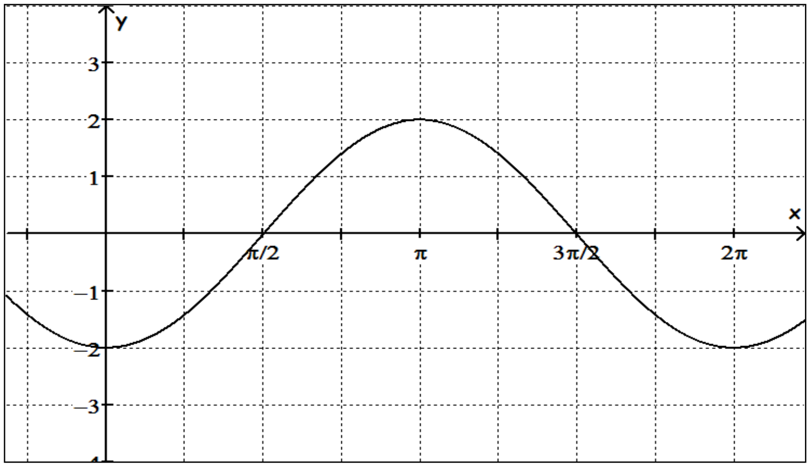


3. For each graph below,

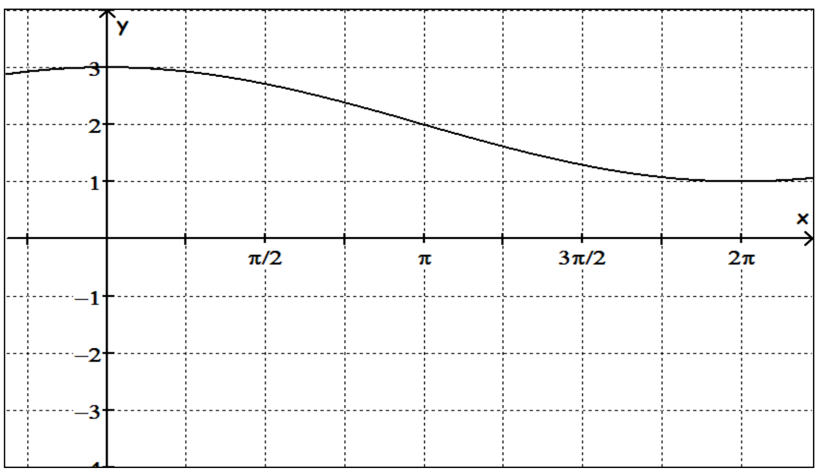
   a) Describe the transformation(s) of $y = \cos x$ that produced the graph.

   b) Give the equation of the transformed function.

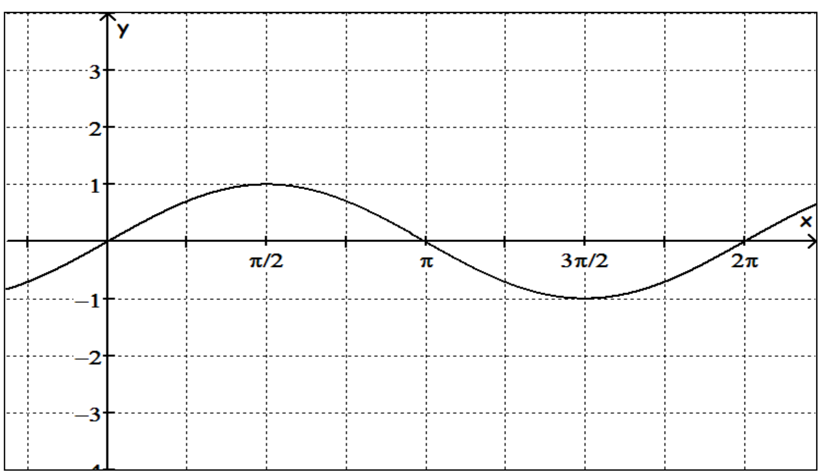
1)

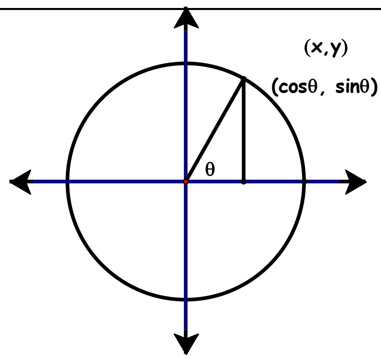


2)



3)

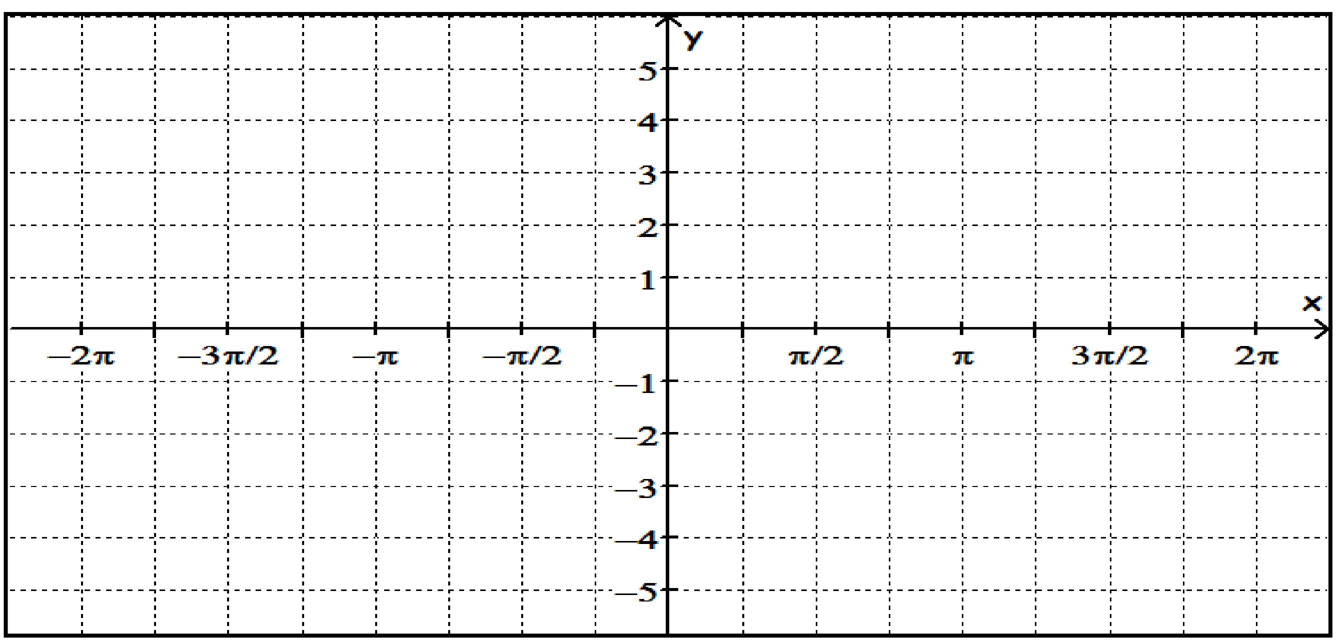




Recall that in a unit circle the *x* and *y* coordinates of points on the circle correspond to the values of $\cos\theta$ and $\sin\theta$, respectively. Use the definition of tangent to find the exact values of $\tan\theta$.

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{2}}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{\sqrt{2}}{2}$	-1	$-\frac{\sqrt{2}}{2}$	0	$\frac{\sqrt{2}}{2}$	1
$\tan\theta$											

Graph $y = \tan x$ on your calculator using domain $-2\pi \le x \le 2\pi$ and range $-5 \le y \le 5$.
Sketch the results below.



a) What happened to the graph at $x = \frac{\pi}{2}$?
Explain why this is so, and name other values of *x* when this occurs.

b) What is the period of $y = \tan x$?

c) Explain why the values of $\frac{\pi}{5}$ and $\frac{6\pi}{5}$ are the same.

d) How would the graph of $y = -\tan x$ be different from its parent function?
Explain the transformation verbally, then check your conjectures on your calculator.

CHALLENGE

The height of water at the mouth of a certain river varies during the tide cycle. The time in hours since midnight, *t*, and the height in feet, *h*, are related by the equation

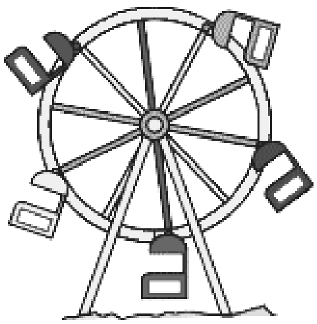
$$h = 15 + 7.5\cos\frac{2\pi}{12}t$$

a) What is the length of a period modeled by this equation?

b) When is the first time the height of the water is 11.5 feet?

c) When will the water be at that height again?

At a fair you board a Ferris wheel whose bottom car is 3 feet above ground. The radius of the wheel is 10 feet and the Ferris wheel makes 3 revolutions per minute.



a) How long does it take the wheel to make one complete revolution?

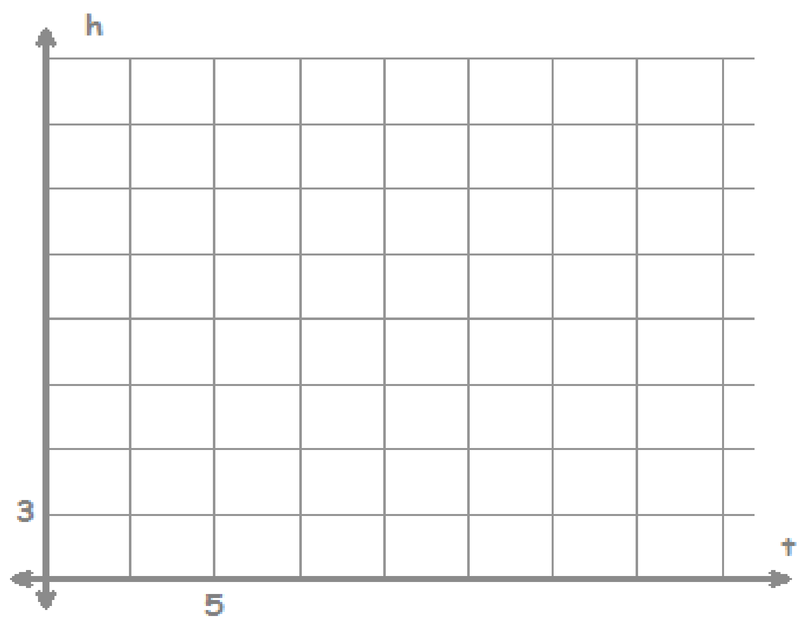
b) If the wheel turns counter-clockwise and you boarded at the bottom, what would be your height at 5 secs?

c) At what time would you first reach the maximum height of the wheel?

d) At what height would you be after 15 seconds? 20 seconds?

e) Complete the table for height in feet (h) versus time in seconds (t) for the first revolution. If height varies sinusoidally with time, sketch one cycle of your graph.

t	h
0	
5	
10	
15	
20	



f) What is the period of your graph?

g) Find the frequency. Show your work.

h) Where is the midline of your graph? Which parameter in the equation has this value?

i) Explain in words the meaning of the amplitude of a sine or cosine curve.

What is the amplitude of this curve?

j) Write an equation that models your graph.

k) Use your calculator to check to see that your equation is accurate. Check the table as well as the graph. Remember to change your window to match the one given in the problem.

Show all work.

1. A person riding a Ferris wheel at a local fair makes one complete trip around in 10 minutes. Their height can be modeled using a sine function of the form y = A sin(Bt) + C, where t is the amount of time the person has been traveling, in minutes. Which of the following must be the value of B?

(1) 10 (2) $\frac{1}{20}$ (3) 10π (4) $\frac{\pi}{5}$

2. The volume of water in a tank varies periodically. At t = 0 it is at its maximum of 650 gallons and at t = 5 it is at its minimum of 120 gallons. Which of the following functions would best model the volume of water in this tank as a function of time in hours?

(1) $V = 265\cos\left(\frac{2\pi}{10}t\right) + 385$

(2) $V = -770\sin(10t) + 385$

(3) $V = -385\cos(5t) + 265$

(4) $V = 265\sin\left(\frac{\pi}{10}t\right) + 770$

3. A person's height, in feet above ground, on a Ferris wheel can be modeled using the equation , $h(t) = -45\cos\left(\frac{\pi t}{7}\right) + 52$ where t is the time the rider has been on the wheel in minutes.

What is the maximum height the rider reaches and the time it takes to first reach this height if they get on at t = 0. Explain how you arrived at your answer.

The table below reviews the rules for exponents that you should remember from Algebra.

ConceptSummary Properties of Exponents

For any real numbers x and y, integers a and b:

Property	Definition	Examples
Product of Powers	$x^a \cdot x^b = x^{a+b}$	$3^2 \cdot 3^4 = 3^{2+4}$ or 3^6 $p^2 \cdot p^9 = p^{2+9}$ or p^{11}
Quotient of Powers	$\frac{x^a}{x^b} = x^{a-b}, x \neq 0$	$\frac{9^5}{9^2} = 9^{5-2}$ or 9^3 $\frac{b^6}{b^4} = b^{6-4}$ or b^2
Negative Exponent	$x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-a}} = x^a, x \neq 0$	$3^{-5} = \frac{1}{3^5}$ $\frac{1}{b^{-7}} = b^7$
Power of a Power	$(x^a)^b = x^{ab}$	$(3^3)^2 = 3^{3 \cdot 2}$ or 3^6 $(d^2)^4 = d^{2 \cdot 4}$ or d^8
Power of a Product	$(xy)^a = x^a y^a$	$(2k)^4 = 2^4k^4$ or $16k^4$ $(ab)^3 = a^3b^3$
Power of a Quotient	$\left(\frac{x}{y}\right)^a = \frac{x^a}{y^a}, y \neq 0$, and $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^a$ or $\frac{y^a}{x^a}, x \neq 0, y \neq 0$	$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$ $\left(\frac{a}{b}\right)^{-5} = \frac{b^5}{a^5}$
Zero Power	$x^0 = 1, x \neq 0$	$7^0 = 1$

Simplify. Assume that no variable equals 0.

1. $b^4 \cdot b^3$

2. $c^5 \cdot c^2 \cdot c^2$

3. $a^{-4} \cdot a^{-3}$

4. $x^5 \cdot x^{-4} \cdot x$

5. $(2x)^2(4y)^2$

6 $-2gh(g^3h^5)$

7. $10x^2y^3(10xy^8)$

8. $\frac{24wz^2}{3w^3z^5}$

9. $\frac{-6a^4bc^8}{36a^7b^2c}$

10. $\frac{-10pt^4r}{-5p^3t^2r}$

11. $n^5 \cdot n^2$

12. $y^7 \cdot y^3 \cdot y^2$

13. $t^9 \cdot t^{-8}$

14. $x^{-4} \cdot x^{-4} \cdot x^4$

15. $(2f^4)^6$

16. $(-2b^{-2}c^3)^3$

17. $(4d^2t^5v^{-4})(-5dt^{-3}v^{-1})$

18. $8u(2z)^3$

19. $\frac{12m^8y^6}{-9my^4}$

20. $\frac{-6n^5x^3}{18nx^7}$

21. $\frac{-27x^3(-x^7)}{16x^4}$

22. $\left(\frac{2}{3r^2t^3z^6}\right)^2$

23. $-(4w^{-3}z^{-5})(8w)^2$

24. $(m^4n^6)^4(m^3n^2p^5)^6$

25. $\left(\frac{3}{2}d^{-2}f^4\right)^4\left(-\frac{4}{3}d^5f\right)^3$

26. $\left(\frac{2x^3y^2}{-x^2y^5}\right)^{-2}$

27. $\frac{(3x^{-2}y^3)(5xy^{-8})}{(x^{-3})^4y^{-2}}$

28. $\frac{-20(m^2v)(-v)^3}{5(-v)^2(-m^4)}$

MULTIPLICATION:

- Like bases: Keep the base the same add the exponents.
  $x^a \bullet x^b = x^{a+b}$
- Coefficients and like bases: Multiply coefficients. Keep the base the same add the exponents.

DIVISION:

- Like bases: Keep the base, subtract the exponents.
  $x^a \div x^b = x^{a-b}$
- Coefficients and like bases: Divide the coefficients. Keep the base and subtract the exponents
  $24x^a \div 3x^b = 8x^{a-b}$

POWERS

- Keep the base; multiply exponents.
  $\left(a^3b^5\right)^3 = a^9b^{15}$
- Remember, if there is a coefficient, its exponent must also be multiplied.
  $\left(4a^3 3b^5\right)^3 = 64a^9 27b^{15}$

ZERO EXPONENT

- Anything raised to the zero power is equal to zero.
  $4^0 = 1$
  $a^0 = 1$ where $a \neq 0$

NEGATIVE EXPONENT

- $x^{-1} = \frac{1}{x}$ where $x \neq 0$
  $\frac{1}{x^{-2}} = x^2$ where $x \neq 0$

FRACTIONAL EXPONENTS

- $x^{\frac{m}{n}} = \sqrt[n]{x^m}$ where $n \neq 0$, $x > 0$ when n is even

EXAMPLES:

1. $x^2 \bullet x^7 \bullet x^{-3}$
2. $a^5 \div a$
3. $x^b \bullet x$
4. $5^a \bullet 5^b$
5. $\frac{6^c}{6^b}$
6. $\frac{\left(7^3\right)^4}{7^{10}}$
7. $\frac{3 \bullet 7^a}{7^a}$
8. $\left(\frac{1}{2}x^2\right)^3$
9. $\frac{-x^5}{(-x)^4}$
10. $\frac{\left(x^3y\right)^3}{xy^3}$
11. $\frac{x^{10}}{x^{-1}}$
12. $\frac{9x^{-5}}{3x^{-8}}$
13. $10^{-2}(5)^2$
14. $25^{1/2}$
15. $4^{-3/2}$
16. $\left(8^0 + 8^1\right)^{1/2}$
17. $8^{\frac{1}{3}} \bullet 8^{\frac{-2}{3}}$
18. $\left(\frac{4}{25}\right)^{\frac{1}{2}}$
19. $27^{\frac{-2}{3}}$
20. $\left(\frac{27}{64}\right)^{\frac{-2}{3}}$

SOLVING EXPONENTIAL EQUATIONS

- Get a common base.
- If the base is the same and the two expressions are equal, then their exponents have to be equal.

$5^{2+1} = 5^3$

$2 + 1 = 3$ (true)   SO

$5^{x+1} = 5^4$

$x + 1 = 4$

$x = 3$

WHEN THE BASES ARE NOT THE SAME

- Change either the right or left side so that they have the same base. To do this, rewrite as an equivalent expression with the same base.

$4^x = 2^{16}$

$\left(2^2\right)^x = 2^{16}$

$2^{2x} = x^{16}$

$2x = 16$

$x = 8$

$9^{x+1} = 27^x$

$\left(3^2\right)^{(x+1)} = \left(3^3\right)^x$

$3^{2x+2} = 3^{3x}$

$2x + 2 = 3x$

$2 = x$

$\left(\frac{1}{4}\right)^x = 8^{(1-x)}$

$\left(2^{-2}\right)^x = \left(2^3\right)^{(1-x)}$

$2^{-2x} = 2^{3-3x}$

$-2x = 3 - 3x$

$x = 3$

Solving a Power Equation:

$x^{\frac{2}{3}} = 25$

$\left(x^{\frac{2}{3}}\right)^{\frac{3}{2}} = (25)^{\frac{3}{2}}$

$x = \left(\sqrt{25}\right)^3$

$x = 125$

1. $4^{x+2} = 4^3$
2. $3^{2x-1} = 3^{x+2}$
3. $\left(\frac{1}{2}\right)^x = 8^{2-x}$
4. $32^x = 4$
5. $5^{3x} = 25^{x+1}$
6. $x^{\frac{5}{3}} = 32$
7. $3^{x^2-3} = 3^{2x}$
8. $\left(\frac{1}{3}\right)^{1-x} = 9^x$
9. $5^{x-1} = 125$
10. $4x^{1/3} = 12$

GRAPHING EXPONENTIAL FUNCTIONS

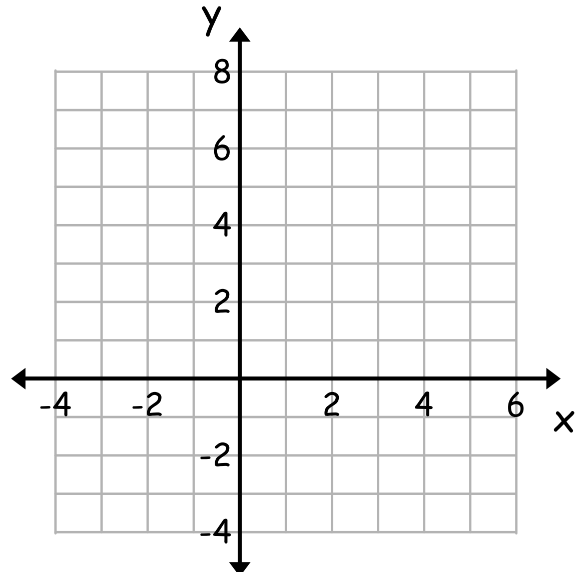
$y = a \cdot b^{x}$

Use your graphing calculator to complete the exploration below.
Complete all questions, tables and graphs carefully.

To start the exploration let a = 1 and b = 2 . This will give you a sketch of the exponential function $y = 1 \bullet 2^{x}$. Use your calculator to complete the table and then sketch the graph of the function using the values from the table.

$y = 1 \bullet 2^{x}$

x	y
-2	
-1	
0	
1	
2	
3	



a) Describe the shape of the graph. Does it get steeper or less steep as *x* increases?

b) Does the graph have a *y* intercept? If so, what is it? If not, explain why not.

c) How is the *y* intercept related to your equation?

d) Does the graph have an *x* intercept? If so, what is it? If not, explain why not.

e) Look at your table – is there a pattern? If so, what is it? How is it related to your equation?

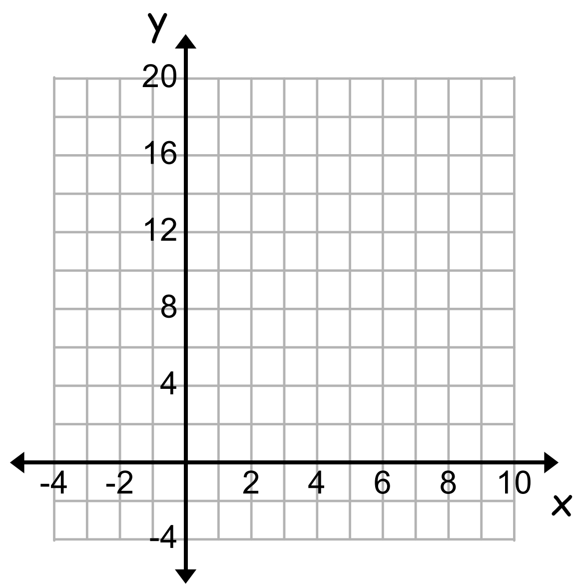
f) Use the pattern that you found in (e) to predict the value of *y* when *x* = 6.
Check your prediction by using the table function on your calculator.

g) If b = 4, what pattern would you expect to see in the table for the function $y = 1 \bullet 4^x$?

What does the "*a*" value represent in this equation?

Complete the table and sketch the graph for $y = 1 \bullet 4^x$.

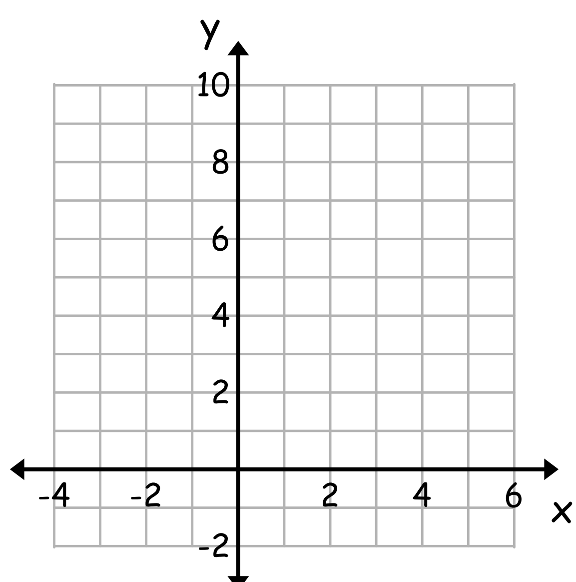
x	y
-1	
0	
1	
2	



Now let's see what happens when $0 < b < 1$.
Change the value of b to .5 or 1/2 and complete the table and sketch the graph.

$y = 1 \bullet .5^x$ or $y = 1 \bullet \left(\frac{1}{2}\right)^x$

x	y
-3	
-2	
-1	
0	
1	
2	



h) How did the graph change?

i) What effect does the "b" value in the equation have on the table?

j) Is the value of "a" still the *y* intercept?

k) Vary the value of "a". Did it do what you expected?
What happens to the graph if "a" changes from positive to negative?

Let's see what you have learned…..

Graph exponential function $y = 1 \bullet 5^x$ on your calculator.
Go to the window and set XMIN to -3, XMAX to 3, and the Xscl to 1.

1. Complete the table:

x	y
-3	
-2	
-1	
0	
1	
2	
3	

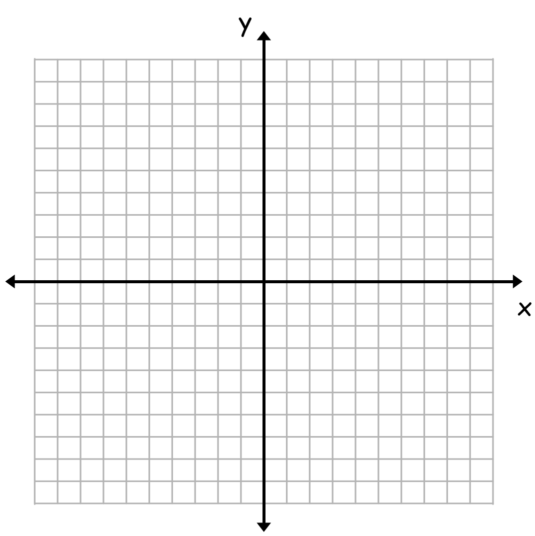
a) What is the "common ratio" of the y values in the table?

b) Complete this sentence:
for $y = 1 \bullet 5^x$, when x increases by 1, y
________________________ .

c) What is the value of the y intercept ?

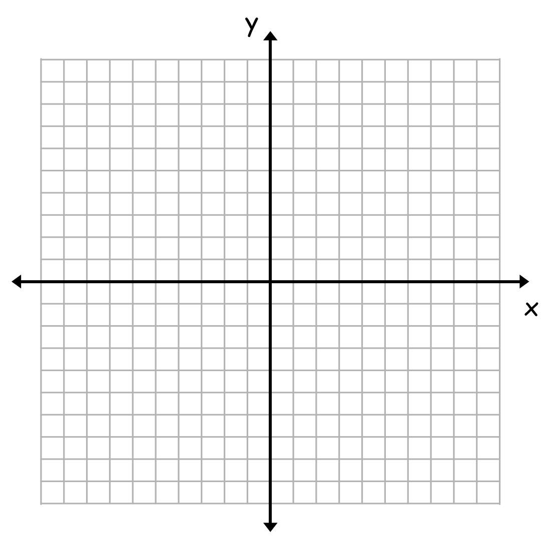
2. Use your calculator to graph an example of $y = a \bullet b^x$ for each case listed below. For each one, state the equation you used and sketch the graph. Also state whether the y values are increasing, decreasing, or constant as you look from left to right across the graph.

a) $a > 0$ *and* $b > 1$



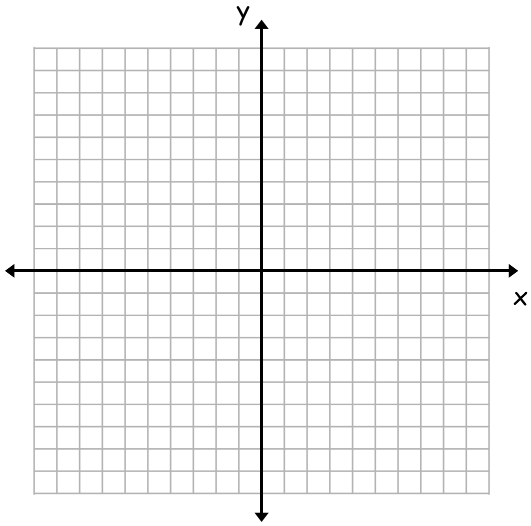
Equation: ________________________

b) $a > 0$, *and* $0 < b < 1$



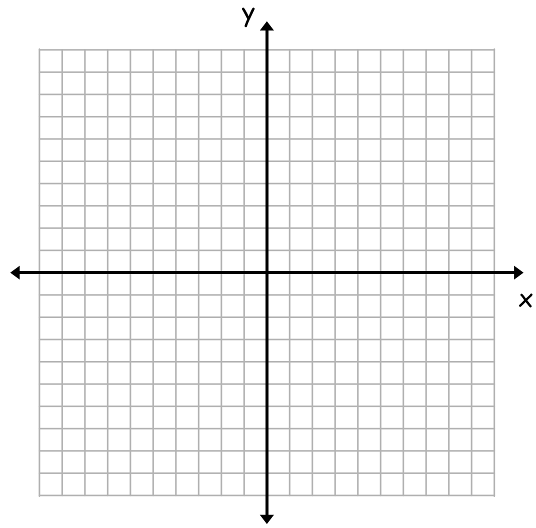
Equation: ________________________

c) $a < 0$ *and* $b > 1$



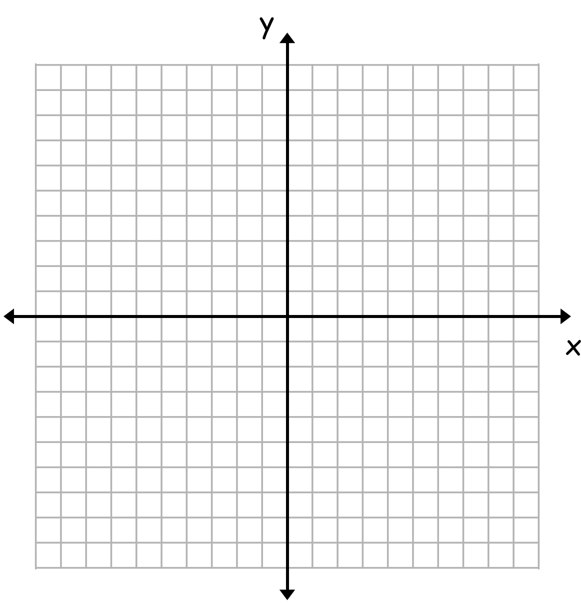
Equation: ______________________

d) $a < 0$ *and* $0 < b < 1$



Equation: _____________________

e) $b = 1$



Now why did that happen? Can you explain?

Equation: _________________________

Module 3 – Exponential and Logarithmic Functions

Solving Exponential Equations Not Requiring Logarithms

Solve each given equation.

1. $4^{3y+1} = 64$

2. $5^{-2s+2} = 5^{-4s}$

3. $9^{2z} \bullet 9^{-4z} = 81$

4. $10^{-3b-3} \bullet 10^{4b} = 10^{-2b}$

5. $5^{2h} \bullet 5^{-3h} = \frac{1}{25}$

6. $2^{-3r-1} \bullet 4 = 2^{-4r}$

7. $729^{3n+1} \bullet 9^{-2n} = 81$

8. $\frac{2^{w}}{2^{2w-1}} = 2^{-4w}$

9. $\left(\frac{1}{9}\right)^{3m-2} \bullet 81^{4m} = \frac{1}{8}$

10. $8^{-2x+3} = 64$

11. $5^{q-2} = 5^{3q}$

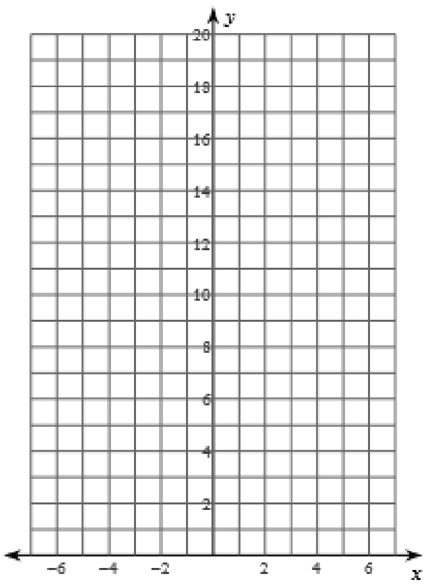
12. $4^{2x} \bullet 4^{-4x} = 16$

13. $6^{2p-3} \bullet 6^{-3p} = 6^{3p}$

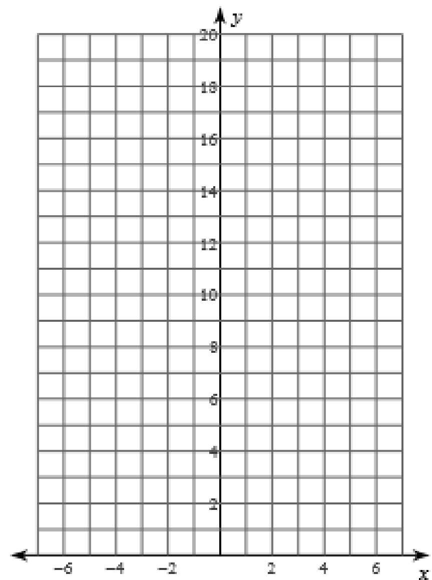
14. $9^{3g} \bullet 9^{-2g} = \frac{1}{729}$

Sketch the graph of each function.

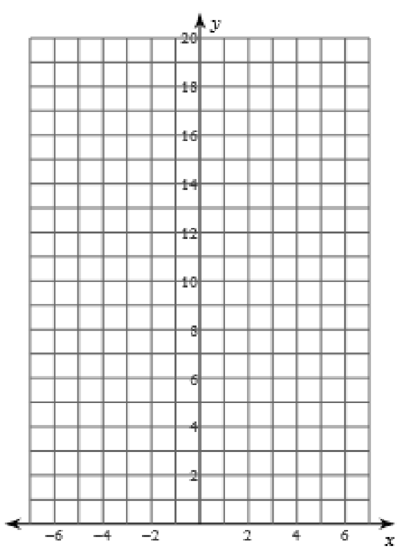
1) $y = 4 \cdot 2^x$



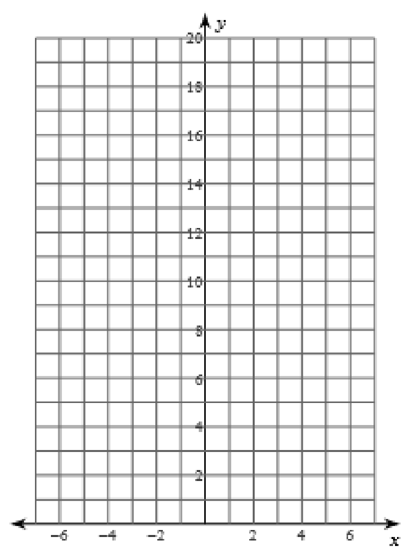
2) $y = 5 \cdot 2^x$



3) $y = 4 \cdot \left(\frac{1}{2}\right)^x$

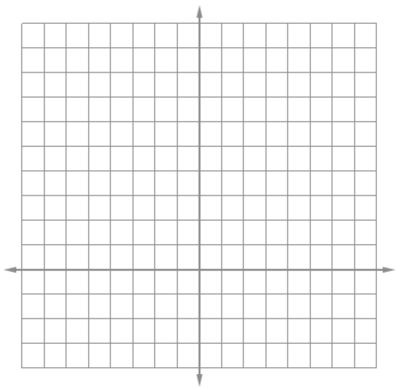
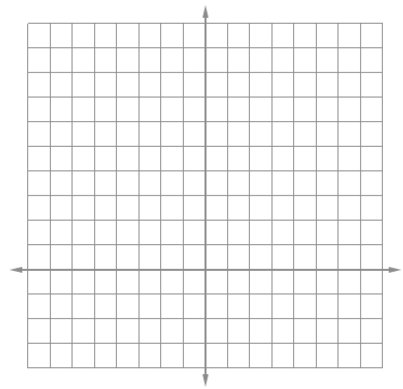
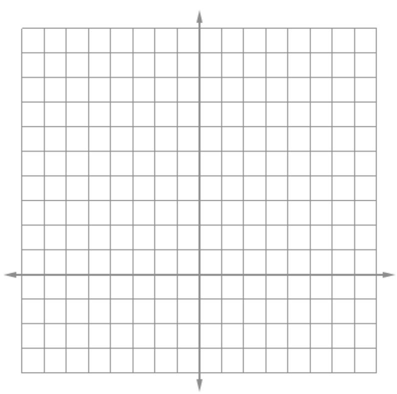
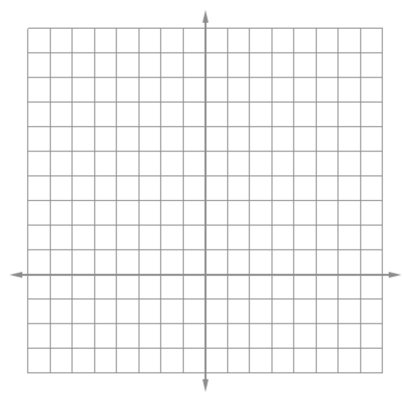
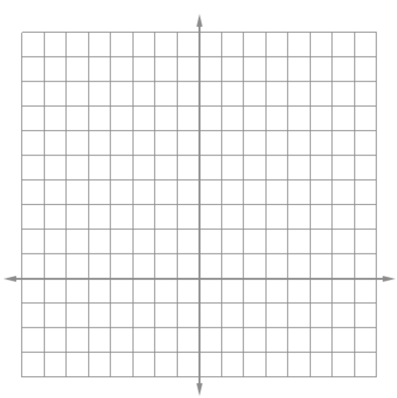
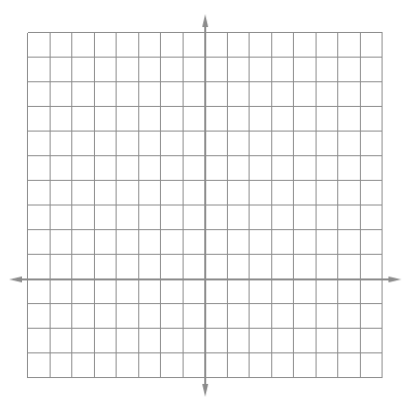


4) $y = 2 \cdot \left(\frac{1}{2}\right)^x$



Module 3 – Exponential and Logarithmic Functions

Graph each of the equations in the table. Note the direction of the graphs and state the percent of change.

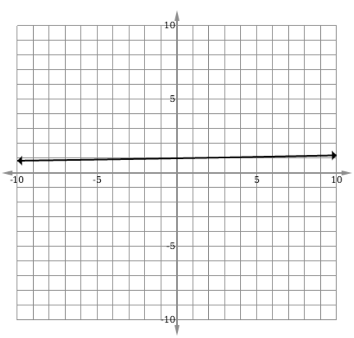
$y=a(1+r)^x$ __________	$y=a(1-r)^x$ __________
1. $y=3^x$ 	2. $y=0.9^x$ 
3. $y=1.9^x$ 	4. $y=0.55^x$ 
5. $y=1.2^x$ 	6. $y=0.4^x$ 

In each situation, state whether the equation is exponential growth or decay, state the percent rate of change and match the letter of the corresponding graph.

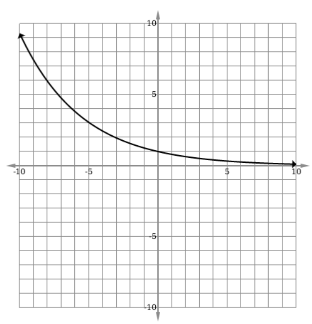
Equation	Growth or Decay	% Rate of Change	Graph
1. $y=(0.80)^x$			
2. $y=(1.5)^x$			
3. $y=4(0.85)^x$			
4. $y=0.3(1.25)^x$			
5. $y=(1.2)^{4x}$			
6. $y=(1.2)^{x/10}$			

Explain how to decide whether the equation is exponential growth or decay.

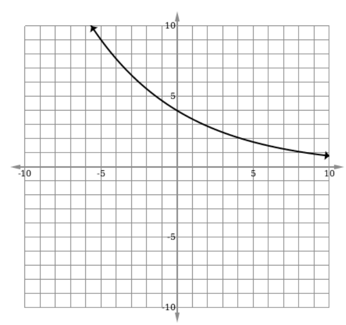
(A)



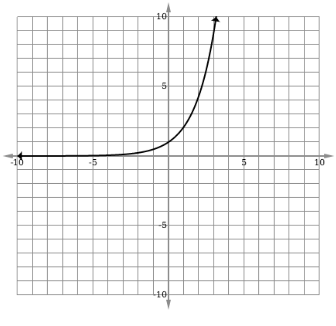
(B)



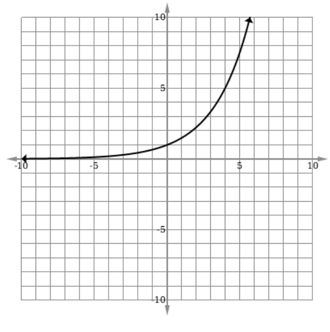
(C)



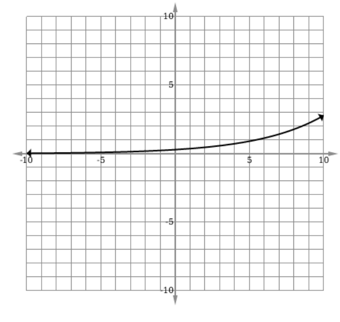
(D)



(E)



(F)



Module 3 – Exponential and Logarithmic Functions

1. A flu outbreak hits an elementary school on Monday, with an initial number of 20 ill students coming to school. The number of ill students than increases by 25% per hour.

   (a) Is this situation an example of exponential growth or decay?

   (b) Write an exponential function to model this flu outbreak.

   (c) How many students will be ill after 6 hours?

2. A total of 50,000 contestants participate in an internet online survivor game. The game randomly kills off 20% of the contestants each day.

   (a) Is this situation an example of exponential growth or decay?

   (b) Write an exponential function to model this game.

   (c) How many contestants are left in the game at the end of one week?

3. A new sports car sells for $35,000. The value of the car decreases by 18% annually. Write an equation to model this situation.

4. At the end of last year, the population of a small town was approximately 75,000 people. The population is growing at the rate of 2.4% each year. In how many years will the population reach 100,000 people?

5. Sean invests $10,000 at an annual rate of 5% compounded continuously, according to the formula $A = Pe^{rt}$, where *A* is the amount, *P* is the principal, $e = 2.718$, *r* is the rate of interest and *t* is time, in years. Determine to the nearest dollar, the amount of money he will have after two years.

   Determine how many years, to the nearest year, it will take for his initial investment to double.

Exponential Modeling with Percent Growth and Decay

1. If $130 is invested in a savings account that earns 4% interest per year, which of the following is closest to the amount in the account at the end of 10 years?

   (1) $218 (2) $192 (3) $168 (4) $324

2. A population of 50 fruit flies is increasing at a rate of 6% per day. Which of the following is closest to the number of days it will take for the fruit fly population to double?

   (1) 18 (2) 12 (3) 6 (4) 28

3. If a radioactive substance is quickly decaying at a rate of 13% per hour approximately how much of a 200 pound sample remains after one day?

   (1) 7.1 pounds (2) 2.3 pounds (3) 25.6 pounds (4) 15.6 pounds

4. A population of llamas stranded on a desert island is decreasing due to a food shortage by 6% per year. If the population of llamas started out at 350, how many are left on the island 10 years later?

   (1) 257 (2) 58 (3) 102 (4) 189

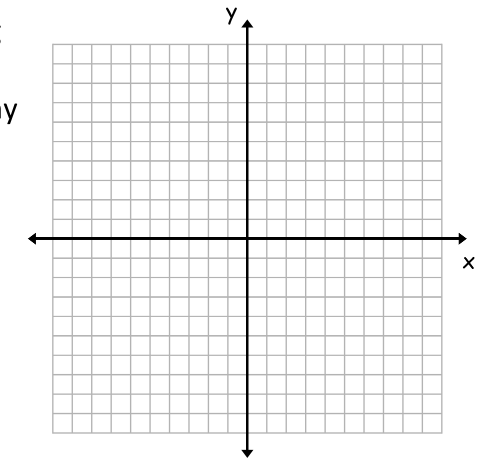
5. Which of the following equations would model a population with an initial size of 625 that is growing at an annual rate of 8.5%?

   (1) $P = 625(8.5)^t$ (2) $P = 625(1.085)^t$
   (3) $P = 1.085^t + 625$ (4) $P = 8.5t^2 + 625$

6. The acceleration of an object falling through the air will decrease at a rate of 15% per second due to air resistance. If the initial acceleration due to gravity is 9.8 meters per second, which of the following equations best models the acceleration *t* seconds after the object begins falling?

   (1) $a = 15 - 9.8t^2$ (2) $a = \frac{9.8}{15t}$
   (3) $a = 9.8(1.15)^t$ (4) $a = 9.8(0.85)^t$

7. Red Hook has a population of 6,200 people and is growing at a rate of 8% per year. Rhinebeck has a population of 8,750 and is growing at a rate of 6% per year. In how many years, to the nearest year, will Red Hook have a greater population than Rhinebeck? Show the equation or inequality you are solving and solve it graphically.

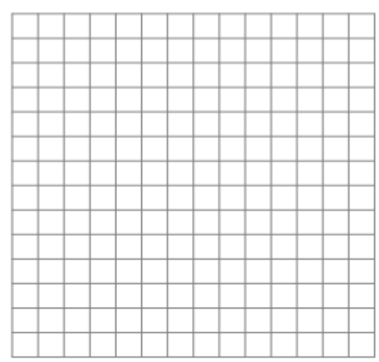


8. A warm glass of water, initially at 120 degrees Fahrenheit, is placed in a refrigerator at 34 degrees Fahrenheit and its temperature is seen to decrease according to the exponential function:

$$T(h) = 86(0.83)^h + 34$$

(a) Verify that the temperature starts at 120 degrees Fahrenheit by evaluating T(0).

(b) Using your calculator, sketch a graph of T below for all values of *h* on the interval $0 \leq h \leq 24$. Be sure to label your y-axis and y-intercept.



(c) After how many hours will the temperature be at 50 degrees Fahrenheit? State your answer to the nearest *hundredth* of an hour. Illustrate your answer on the graph you drew in (b).

9. A house purchased 5 years ago for $100,000 was just sold for $135,000. Assuming exponential growth, approximate the annual growth rate, to the *nearest percent*.

10. Last year, the total revenue for Home Style, a national restaurant chain, increased 5.25% over the previous year. If this trend were to continue, which expression could the company's chief financial officer use to approximate their monthly percent increase in revenue? [Let *m* represent months.]

1) $(1.0525)^{m}$
2) $(1.0525)^{\frac{12}{m}}$
3) $(1.00427)^{m}$
4) $(1.00427)^{\frac{m}{12}}$

11. An equation to represent the value of a car after *t* months of ownership is $v = 32,000(0.81)^{\frac{t}{12}}$. Which statement is *not* correct?

1) The car lost approximately 19% of its value each month.
2) The car maintained approximately 98% of its value each month.
3) The value of the car when it was purchased was $32,000.
4) The value of the car 1 year after it was purchased was $25,920.

12. A payday loan company makes loans between $100 and $1000 available to customers. Every 14 days, customers are charged 30% interest with compounding. In 2013, Remi took out a $300 payday loan. Which expression can be used to calculate the amount she would owe, in dollars, after one year if she did not make payments?

1) $300(.30)^{\frac{14}{365}}$
2) $300(1.30)^{\frac{14}{365}}$
3) $300(.30)^{\frac{365}{14}}$
4) $300(1.30)^{\frac{365}{14}}$

13. In 2010, the population of New York State was approximately 19,378,000 with an annual growth rate of 1.5%. Assuming the growth rate is maintained for a large number of years, which equation can be used to predict the population of New York State *t* years after 2010?

1) $P_t = 19,378,000(1.5)^t$
2) $P_0 = 19,378,000$
   $P_t = 19,378,000 + 1.015P_{t-1}$
3) $P_t = 19,378,000(1.015)^{t-1}$
4) $P_0 = 19,378,000$
   $P_t = 1.015P_{t-1}$

14. Seth's parents gave him $5000 to invest for his 16th birthday. He is considering two investment options. Option *A* will pay him 4.5% interest compounded annually. Option *B* will pay him 4.6% compounded quarterly. Write a function of option *A* and option *B* that calculates the value of each account after *n* years. Seth plans to use the money after he graduates from college in 6 years. Determine how much more money option *B* will earn than option *A* to the *nearest cent*. Algebraically determine, to the *nearest tenth of a year*, how long it would take for option *B* to double Seth's initial investment

15. A student studying public policy created a model for the population of Detroit, where the population decreased 25% over a decade. He used the model $P = 714(0.75)^d$, where P is the population, in thousands, d decades after 2010. Another student, Suzanne, wants to use a model that would predict the population after y years. Suzanne's model is best represented by

1) $P = 714(0.6500)^y$
2) $P = 714(0.8500)^y$
3) $P = 714(0.9716)^y$
4) $P = 714(0.9750)^y$

16. The function $p(t) = 1010e^{0.03922t}$ models the population of a city, in millions, t years after 2010. As of today, consider the following two statements:

    I. The current population is 110 million.
    II. The population increases continuously by approximately 3.9% per year.

    This model supports

    1) I, only
    2) II, only
    3) both I and II
    4) neither I nor II

17. Jasmine decides to put $100 in a savings account each month. The account pays 3% annual interest compounded monthly. How much money, S, will Jasmine have after one year?

    1) $S = 100(1.03)^{12}$
    2) $S = \frac{100 - 100(1.0025)^{12}}{1 - 1.0025}$
    3) $S = 100(1.0025)^{12}$
    4) $S = \frac{100 - 100(1.03)^{12}}{1 - 1.03}$

18. Jim is looking to buy a vacation home for $172,600 near his favorite southern beach. The formula to compute a mortgage payment, M, is $M = P \bullet \frac{r(1+r)^N}{(1+r)^N - 1}$ where P is the principal amount of the loan, r is the monthly interest rate, and N is the number of monthly payments. Jim's bank offers a monthly interest rate of 0.305% for a 15 year mortgage.

    With no down payment, determine Jim's mortgage payment, rounded to the *nearest dollar*.

    Algebraically determine and state the down payment, rounded to the *nearest dollar*, that Jim needs to make in order for his mortgage payment to be $1100.

The table below shows the number of Starbucks stores yearly from 1996 to 2006.
Use $x = 1$ for the year 1996.

Year	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
# Stores	1015	1412	1886	2498	3501	4709	5886	7225	8569	10241	12440

a) Create a scatterplot of the number of stores vs. time *in years since 1996.* Label axes and give window dimensions.

b) What appears to be the form of this data?

c) Use your calculator to find a regression equation to model this data. Define your variables, and round decimals to the nearest thousandth.

d) If this pattern continues, what would be the number of Starbucks stores at the end of fiscal 2008? Show your work.

e) Use your model to "predict" the number of Starbucks stores that were in existence in 1995.

Module 3 – Exponential and Logarithmic Functions

1. The population of Jamestown has been recorded for selected years since 2000. The table below gives these populations. Use $x=0$ to represent the year 2000.

(a) Determine an exponential regression equation, of the form $y=a(b)^x$, where *x* represents the number of years since 2000 and *y* represents the population. Round *a* to the nearest integer and *b* to the nearest thousandth.

(b) By what percent does your exponential model predict the population is increasing per year?

(c) Determine the number of years, to the nearest year, for the population to reach 20 thousand.

Year	2002	2004	2005	2007	2009
Population	5564	6121	6300	6312	7422

2. The infiltration rate of a soil is the number of inches or water per hour it can absorb. Hydrologists studied one particular soil and found its infiltration rate decreases exponentially as a rainfall continues.

Time, *t* (hours)	0	1.5	3.0	4.5	6.0
Infiltration Rate, *I* (inches per hour)	5.3	3.1	2.4	1.6	0.7

Determine an exponential model that best fits this data set. Round *a* and *b* to the nearest hundredth. Use your equation to determine the time when the infiltration rate reaches 0.8 inches per hour. Round your answer to the nearest tenth of an hour. Show your work.

Composition of Functions

Composition: use the output (result) of one function to evaluate a second function.

ReadingMath

Composition of Functions

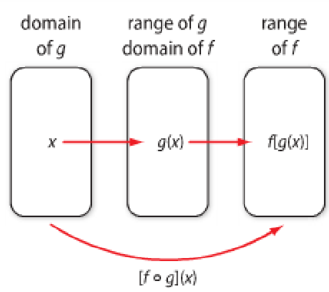
The composition of f and g, denoted by $f \circ g$ or $f[g(x)]$, is read *f of g*.

KeyConcept Composition of Functions

Words Suppose f and g are functions such that the range of g is a subset of the domain of f. Then the composition function $f \circ g$ can be described by

$$[f \circ g](x) = f[g(x)].$$

Model



f = {(1, 8), (0, 13), (15, 11), (14, 9)}

g = {(8, 15), (5, 1), (10, 14), (9, 0)}

Find:

1. **f(g(8)) =**

2. **g(f(15)) =**

3. **f(g(10)) =**

4. **g(f(14)) =**

If f(x) = 2x – 5 and g(x) = 4x find the following:

1. **f(g(5)) =**

2. **G(f(-2)) =**

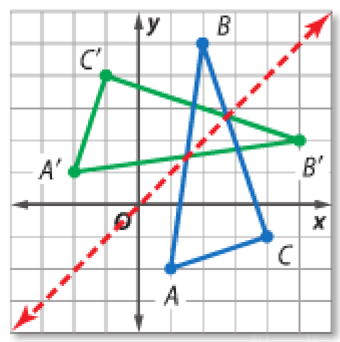
3. **f(g(4a)) =**

4. **g(f(2a – 5)) =**

A new car dealer is discounting all new cars by 12%. At the same time, the manufacturer is offering a \$1500 rebate on all new cars. Ms. Hillig is buying a car that is priced \$24,500. Will the final price be lower if the discount is applied before the rebate or if the rebate is applied before the discount?

<u>Inverses</u>

Two relations are INVERSES if and only if whenever One relation contains the element (a, b), the other Relation contains the element (b, a).



Triangle ABC with vertices A(1, -2), B(2, 5), and C(4, -1) is Reflected over the line y=x. What are the vertices of A′, B′, and C′?

Module 3 – Exponential and Logarithmic Functions

Find the inverse of $f(x) = 2x - 5$

Verify with compositions

$f(f^{-1}(x)) =$

$f^{-1}(f(x)) =$

Determine whether each pair of functions are inverse functions. Explain your reasoning.

1. **$f(x) = 3x + 9$** **$g(x) = 1/3x - 3$**

2. **$f(x) = 4x^2$** $g(x) = 2\sqrt{x}$

Module 3 – Exponential and Logarithmic Functions

Practice:

Find the inverse of each relation.

1. {(-8, 6), (6, 2), (7, -3)}
2. {(8, -1), (-8, -1), (-2, -8), (2, 8)}
3. {(1, 5), (2, 6), (3, -7), (4, 8), (5, -9)
4. {(3, 0), (5, 4), (7, -8), (9, 12), (11, 16)}

Find the inverse of each function.

5. f(x) = x + 2
6. g(x) = 5x
7. $h(x) = \frac{x-4}{3}$
8. $f(x) = \frac{-5}{3}x - 8$

Determine whether each pair of functions are inverse functions. Write yes or no. Show all work.

9. f(x) = 2x + 3
   g(x) = 2x – 3
10. f(x) = 4x + 6
    $g(x) = \frac{x - 6}{4}$
11. f(x) = -6x
    $g(x) = \frac{1}{6}x$
12. $f(x) = \frac{1}{2}x + 5$
    g(x) = 2x - 10

Solving Exponential Equations

If $b^x = b^n$, then ________

KNOW YOUR POWERS!!	
$2^2 =$	$3^2 =$
$2^3 =$	$3^3 =$
$2^4 =$	$3^4 =$
$2^5 =$	$5^2 =$
$2^6 =$	$5^3 =$
$\frac{1}{3} =$	$\sqrt{a} =$
$\frac{1}{9} =$	$\sqrt[3]{a} =$

1. $5^x = 125$

2. $8^x = 4^{x+1}$

3. $2^{x+5} = \frac{1}{8}$

4. $4^{2x} = 64$

5. $8^{2x} = 32^{x+2}$

6. $36^x = \sqrt{6}$

7. $25^x = 125$

8. $4^x = \dfrac{1}{16}$

9. $3^{3x-1} = 9^{x-3}$

10. $\sqrt[3]{5} = 25^x$

11. $4^{3x+1} = 8^{x-1}$

12. $3^{3x-2} = 27^{3x+2}$

13. $4^x = 32$

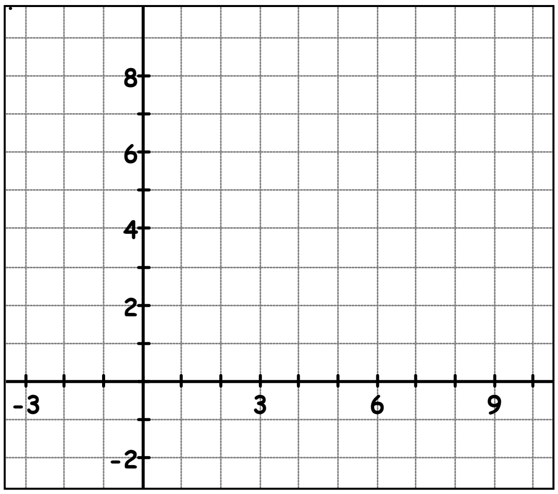
14. Using your calculator and find the value of x to the fourth decimal place.

    (a) $10^x = 846$

    (b) $2^x = 44$

    (c) $3^x = 1.162$

Sketch the graph and complete the table for the function $y = 2^x$.



x	y
-2	
-1	
0	
1	
2	
3	

Complete the table for the inverse of the function $y = 2^x$.

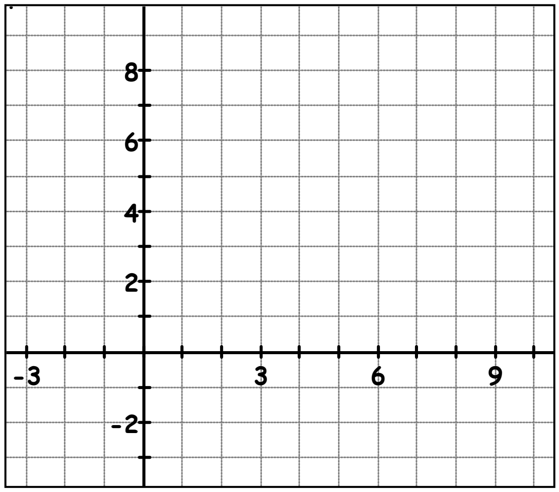
Sketch the graph of this inverse on the axes above.

x	y

Evaluate expressions with logarithms:

Writing logarithmic equations in exponential form:

1. Sketch the graph and complete the table for the function $y = \left(\frac{1}{2}\right)^x$.



x	y
-3	
-2	
-1	
0	
1	
2	

2. Complete the table for the inverse of the function $y = \left(\frac{1}{2}\right)^x$.

   Sketch the graph of this inverse on the axes above.

   Write the inverse function in:

   Exponential form:

   Logarithmic form:

x	y

3. Evaluate the following expressions:

   a) $\log_{\frac{1}{2}} 8 =$

   b) $\log_{\frac{1}{2}} 1 =$

   c) $\log_{\frac{1}{2}} \frac{1}{2} =$

   d) $\log_{\frac{1}{2}} 4 =$

4. Write each of your equations from #3 in exponential form.

   a)

   b)

   c)

   d)

Module 3 – Exponential and Logarithmic Functions

Change exponential form to logarithmic form.

1. $2^3 = 8$
2. $10^{-3} = \frac{1}{1000}$
3. $7^0 = 1$
4. $9^{\frac{1}{2}} = 3$
5. $5^4 = 625$

Change logarithmic form to exponential form.

1. $\log_{10} 100 = 2$
2. $\log_4 64 = 3$
3. $\log_7 7 = 1$
4. $\log_3 \frac{1}{9} = -2$
5. $\text{Log}_{\frac{1}{2}} 1 = 0$

Fill in the blank.

1. $\log_{10} 1000 =$ _____
2. $\log_2$ _____ $= 3$
3. $\log_4$ _____ $= 3$
4. log__ 9 = -2
5. log___ 1 = 0
6. $\log_6 36 =$ ______

$\log_b x$ means What power of ____ gives you ____ ?

Solving logarithmic equations:

1. $\log_5(3x+1)=2$
2. $\log_2 16 = 3x+1$
3. $\log_{27} 9 = x$
4. $\log_9(4x) = -1$
5. $\log_2 32 = 8x - 4$
6. $\log_3(3x-5) = \log_3(4x-9)$
7. $\log_2(5x+3) = \log_2(2x+10)$

Find the value using your calculator. Round to 3 decimal places. Check it!

1. $\log_8 43$
2. $\log_5 872$
3. $\log_6 \frac{2}{3}$
4. $\log 1000$
5. $\log_{52} 38$

Solve the following equations using your calculator.

6. $5^x = 162$
7. $7^x = 1{,}493$
8. $6^x = \frac{1}{215}$
9. $2.3^x = 57{,}862$
10. $8^x = 3.27$
11. $12^{x+3} = 16$
12. $10^{2x} = 1873$
13. $\left(\frac{1}{2}\right)^{x-4} = 78$
14. $\log_8 x = 1.6$
15. $5^{2x-3} = 783$

Module 3 – Exponential and Logarithmic Functions

Solve each equation-NO CALCULATOR!

1. $\log_6 x = 2$

2) $\log_3(2x+1) = 4$

3) $\log_2\left(\frac{x}{8}\right) = -4$

4) $\log_6 36 = 2x$

5) $\log_{16} 32 = x$

6) $\log_4(2x - 7) = \log_4(7x - 10)$

7) $\log_5 1 = x + 14$

8) $\log_2 32 = x + 1$

9) $\log_5 \sqrt{5} = x$

Solve using a calculator

10) $7^x = 20$

11) $12^{k-2} = 20$

12) $\log_7 x = 3.6$

13) $6^{2x+3} = 1{,}423$

14) $\log x = 6.24$

-14	4	½	½	.526	3/5	1	5/4	1.540	3.206	36	40	1102.435	1737800.829

Modeling with Logarithms

1. The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$ where x represents the amplitude of the seismic wave causing ground motion.

   How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 8 as an aftershock with a Richter scale rating of 5?

2. The magnitude of an earthquake is measured on a logarithmic scale called the Richter scale. The magnitude M is given by $M = \log_{10} x$ where x represents the amplitude of the seismic wave causing ground motion.

   In 1906, San Francisco was almost completely destroyed by a 7.8 magnitude earthquake. In 1911, an earthquake estimated at a magnitude 8.2 occurred along the New Madrid fault in the Mississippi River Valley. How many times greater was the New Madrid earthquake than that of San Francisco?

3. The first key on a piano keyboard corresponds to a pitch with a frequency of 27.5 cycles per second. With every successive key, going up the black nd white keys the pitch multiplies by a constant. The formula for the **frequency** of the pitch sounded when the **nth** note up the keyboard is played is given by

$$n = 1 + 12\log_2 \frac{f}{27.5}$$

a) A note has a frequency of 220 cycles per second. How many notes up the piano keyboard is this?

b) Another pitch on the keyboard has a frequency of 880 cycles per seconds. After how many notes up the keyboard will this be found?

Module 3 – Exponential and Logarithmic Functions

Rewrite each of the following in the form $\log_b(x) = L$	Rewrite each of the following in the form $b^L = x$.
1. $16^{\frac{1}{4}} = 2$ ________________ 2. $1000 = 10^3$ ________________ 3. $r = b^k$ ________________	4. $4 = \log_5(625)$ ________________ 5. $\log_{10}(0.01) = -2$ ________________ 6. $\frac{1}{2} = \log_{81}(9)$ ________________
Consider the logarithms base 2. For each logarithmic expression below, either calculate the value of the expression or explain why the expression does not make sense.	Consider the logarithms base 3. For each logarithmic expression below, either calculate the value of the expression or explain why the expression does not make sense.
1. $\log_2(1024)$ ________________ 2. $\log_2(128)$ ________________ 3. $\log_2(\sqrt{8})$ ________________ 4. $\log_2(0)$ ________________ 5. $\log_2\left(-\frac{1}{32}\right)$ ________________	5. $\log_3(243)$ ________________ 6. $\log_3(27)$ ________________ 7. $\log_3\left(-\frac{1}{3}\right)$ ________________ 8. $\log_3(1)$ ________________ 9. $\log_3(0)$ ________________
Evaluate.	Evaluate.
1. $\log_2(8) + \log_2(4)$ 2. $\log_4(4) + \log_4(16)$ 3. $\log_3(9) + \log_3(9)$	4. $\log_2(32)$ 5. $\log_4(64)$ 6. $\log_3(81)$
Summary:	

Evaluate.	Evaluate.
1. $\log_2\left(128\right)+\log_2\left(\frac{1}{8}\right)$	6. $\log_2\left(128\cdot\frac{1}{8}\right)$
2. $\log_2\left(\frac{1}{8}\right)+\log_2\left(\sqrt{2}\right)$	7. $\log_2\left(\frac{1}{8}\cdot\sqrt{2}\right)$
3. $\log_2\left(\sqrt{2}\right)+\log_2\left(128\right)$	8. $\log_2\left(\sqrt{2}\cdot 128\right)$
4. $\log_3\left(3\right)+\log_3\left(27\right)$	9. $\log_3\left(3\cdot 27\right)$
5. $\log_6\left(6\right)+\log_6\left(36\right)$	10. $\log_6\left(6\cdot 36\right)$

Evaluate.	Evaluate.
1. $\log_2\left(128\right)-\log_2\left(\frac{1}{8}\right)$	6. $\log_2\left(128 \text{¸} \frac{1}{8}\right)$
2. $\log_2\left(\frac{1}{8}\right)-\log_2\left(\sqrt{2}\right)$	7. $\log_2\left(\frac{1}{8} \text{¸} \sqrt{2}\right)$
3. $\log_2\left(\sqrt{2}\right)-\log_2\left(128\right)$	8. $\log_2\left(\sqrt{2} \text{¸} 128\right)$
4. $\log_3\left(3\right)-\log_3\left(27\right)$	9. $\log_3\left(3 \text{¸} 27\right)$
5. $\log_6\left(6\right)-\log_6\left(36\right)$	10. $\log_6\left(6 \text{¸} 36\right)$

SUMMARY: PRODUCT AND QUOTIENT RULE FOR LOGS

Module 3 – Exponential and Logarithmic Functions

Write the exponential equation in logarithmic form.		Write the logarithmic equation in exponential form.	
1. $3^5 = 243$	2. $\frac{1}{4} = 2^{-2}$	3. $\log_2\left(\frac{1}{4}\right) = -2$	4. $9 = \log_2(512)$

Solve for "x".		
5. $\log_4(x) = 3$	6. $\log_9(x) = -2$	7. $\log_{\sqrt{3}}(x) = 6$
8. $\log_x(32) = 5$	9. $\log_x(27) = \frac{3}{2}$	10. $\log_x\left(\frac{1}{2}\right) = -1$

Evaluate each expression.		
11. $\log_9(1)$	12. $3\log_2(256)$	13. $\log_2(2^3)^7$

Expand each expression.		
14. $\log_b(xy)$	15. $\log_b\left(\frac{2}{3}\right)$	16. $\log_b\left(\sqrt[4]{x^3}\right)$

Write as a single log, (condense the log).		
17. $3\log_b(x)+5\log_b(y)$	18. $\log_3(2)-\log_3(y)$	19. $\log_2(x+3)-\log_2(3)$
20. $\frac{3}{2}\log_5(4)-2\log_5\left(\frac{1}{2}\right)$	21. $3\log_5(2)-\frac{1}{2}\log_5(4)$	22. $2\log\ (3)-\frac{1}{2}\log\ (9)$

Use the properties of expanding and/or condensing logarithms to simplify each expression.		
23. $\log\ (40)-\log\ \left(\frac{2}{5}\right)$	24. $\log_2(160)-\log_2(5)$	25. $\log_6(90)-\log_6(15)$
26. $\log_3(36)+\log_3(108)$	27. $\log_2(80)+\log_2\left(\frac{2}{5}\right)$	28. $\log_6(4)+\log_6(54)$

Solve each logarithmic equation.		
29. $\log\ (1000)=x$	30. $\log_{36}(216)=x$	31. $\log_2(0.125)=x$
32. $\log\ (x+3)=\log\ (x-5)+\log\ (3)$		

Use the log table below to approximate the following logarithms to four decimal places. The first one has been done for you.

x	$\log(x)$
1	0.0000
2	0.3010
3	0.4771
4	0.6021
5	0.6990

x	$\log(x)$
6	0.7782
7	0.8451
8	0.9031
9	0.9542
10	1.0000

EXAMPLE:

$$\log(500) = \log(100 \cdot 5)$$
$$= \log(10^2 \cdot 5)$$
$$= 2\log(10) + \log(5)$$
$$= 2(1) + \log(5) \leftarrow [\text{Look at the table for this}]$$
$$= 2 + 0.6990$$
$$= 2.6990$$

1. $\log(70{,}000)$

2. $\log(0.008)$

3. $\log(20)$

4. $\log(0.00005)$

5. $\log(125)$

6. $\log(14)$

7. $\log(35)$

8. $\log(72)$

9. $\log\left(\frac{1}{64}\right)$

10. $\log(15)$

11. $\log\left(\frac{3}{7}\right)$

12. $\log\left(\sqrt[4]{2}\right)$

Now, use the tables to calculate the following values. Do they appear anywhere else in the table?

13. $\log(2) + \log(4)$

Where else in the table does this answer appear?

14. $\log(6) - \log(2)$

Where else in the table does this answer appear?

15. $\log(2) + \log(5)$

Where else in the table does this answer appear?

Apply properties of logarithms to rewrite the following expressions as a single logarithm or number.		
16. $\frac{1}{2}\log(25)+\log(4)$	17. $\frac{1}{3}\log(8)+\log(16)$	18. $3\log(5)+\log(0.8)$

Apply properties of logarithms to rewrite each expression as a sum of terms involving numbers, $\log(x)$ and $\log(y)$.		
19. $\log(3x^2y^5)$	20. $\log\left(\sqrt{x^7y^3}\right)$	21. $\log\left(\frac{100x^2}{y}\right)$

Let $\log(X)=r, \log(Y)=s$, and $\log(Z)=t$. Express each of the following in terms of *r, s*, and *t*.		
22. $\log\left(\frac{X}{Y}\right)$	23. $\log(YZ)$	24. $\log(X^r)$
25. $\log\left(\sqrt[3]{Z}\right)$	26. $\log(XY^2Z^3)$	27. $\log\left(\sqrt[4]{\frac{Y}{Z}}\right)$

Use the log table below to approximate the following logarithms to four decimal places.

x	$\log(x)$
1	0
2	0.3010
3	0.4771
4	0.6021
5	0.6990
6	0.7782
7	0.8451
8	0.9031
9	0.9542

x	$\log(x)$
10	1.0000
12	1.0792
16	1.2041
18	1.2553
20	1.3010
25	1.3979
30	1.4771
36	1.5563
100	2.0000

1. $\log(6)+\log(6)$
2. $\log(4)+\log(5)$
3. $\log(30)-\log(3)$
4. $\log(36)-\log(9)$

REVIEW:

1. $2^4 = 16^x$

REVIEW:

2. $8^3 = 16^n$

NEW:

3. $10^x = 3$

4. $10^y = 30$

5. $3^x = 5$

6. $3 \cdot 5^x = 21$

7. $10^{x-3} = 25$

8. $2^{x} = 5$

9. $4^{x} = 36$

Discussion:
Look at # 3,4,5,8,9:

CHANGE OF BASE FORMULA FOR LOGARITHMS:

- If x, a, and b are all positive real numbers with $a \neq 1$ and $b \neq 1$, then:
- $\log_b(x) = \dfrac{\log_a(x)}{\log_a(b)}$

1. $5^{2x} = 20$

2. $75 = 10 \cdot 5^{x-1}$

3. $2^{3x} = 16$

4. $3^{4x-2} = 27^{x+2}$

5. $2^{x} = 81$

6. $\log_5(x) = 4$

Rationals	**Irrationals**
$\frac{1}{2} =$	$p =$
$\frac{1}{3} =$	$\sqrt{2} =$
	$e =$

e is a special irrational number that has many applications in math.

A log with of base of e is called a ___________________.

Instead of writing $\log_e x$, we write it as _________.

Remember the big idea: logs and exponentials are inverses!

Find the value:

1. $\log_3 3^k$ 2. $\ln e^2$ 3. $\ln e^{8x}$ 4. $\ln \sqrt{e}$ 5. $\ln \frac{1}{e}$

Solve each equation. Round to 3 decimal places.

6. $e^x = 9$ 7. $e^{2x} = 4.3$ 8. $3e^x = 21$ 9. $e^{\frac{x}{3}} - 4 = -3$

10. $\ln 2x = 4$ 11. $\ln (3x + 2) = 6$ 12. $3 + 2 \ln x = 11$

Module 3 – Exponential and Logarithmic Functions

So you think you can log….(remember mixed practice may seem harder, but it will be worth it!) Check w/ calc!

13) $2e^{x} = 1{,}234$

14) $\log_5(x + 2) = 3.4$

15) $\ln(3x + 1) = \ln(x + 7)$

16) $\ln(3x^2) = 5$

17) $\log 140 = x$

18) $e^{x+1} - 2 = 29$

19) $\ln\left(\frac{x-1}{2}\right) = 5$

20) $4^{x+1} = 64$

21) $3(6)^{x-2} = 27$

22) $\log_2 64 = 3x - 6$

23) $2e^{x} + 3 = 5$

24) $\ln(2x) = -1.8$

Module 3 – Exponential and Logarithmic Functions

1. When John was born, his grandparents deposited \$3000 into a college saving account paying 4% interest compounded continuously. $A = Pe^{rt}$

   a. Assuming there are no deposits or withdrawals from the account, what will the balance be after 10 years?

   b. How long will it take the balance to reach at least \$10,000?

   c. If his grandparents want John to have \$10,000 after 18 years, how much would they need to invest?

2. The value of a car can be modeled by the equation $y = 24{,}000(0.845)^t$ where t is the number of years since the car was purchased.

   a. After how many years will the value of the car be \$10,000?

   b. Use the model to predict the value of the car after 50 years. Is this a reasonable value? Explain.

3. Suppose the population of a country is currently 8,100,000. Studies show this country's population is increasing 2% each year.

   a. What exponential function would be a good model for this country's population?

   b. Use your exponential equation to find how many years it would take this country's population to reach 9 million.

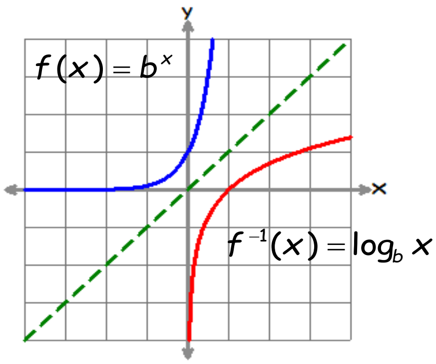
Module 3 – Exponential and Logarithmic Functions

DEFINITION OF A LOGARITHM WITH BASE "B"

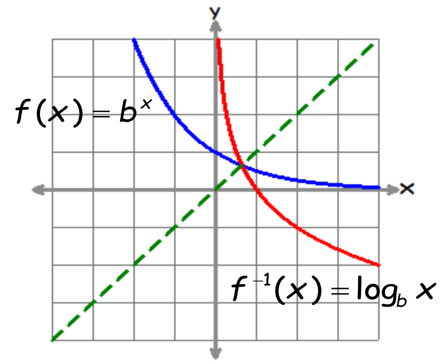
For a > 0 and b > 0

$$\log_b a = x \text{ if and only if } b^x = a$$

Graphs of f and f^{-1} for $b > 1$



Graphs of f and f^{-1} for $0 < b < 1$



SPECIAL LOG VALUES:

LOGARITHM OF 1	$\log_b 1 = 0$ because $b^0 = 1$
LOGARITHM OF BASE B	$\log_b b = 1$ because $b^1 = b$

PROPERTIES OF LOGS:

PRODUCT PROPERTY	$\log_b xy = \log_b x + \log_b y$
QUOTIENT PROPERTY	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
POWER PROPERTY	$\log_b x^n = n\log_b x$

Sigma Notation with Calculator

Find the sum of first 5 terms of he geometric sequence whose explicit formula is $a_n = 3(2)^{n-1}$.

TI 84: Alpha Window/2

$$\sum_{N=1}^{5}\left(3(2)^{N-1}\right)$$

TI 83: 2nd Stat/Math5:sum(
2nd Stat/Ops5:seq(

sum(seq(3(2)^(x-1),x,1,5)

both SUMS are 93

Finding terms or a Specific Term of a Sequence with Calculator

with Explicit Formula:

TI 84: 2nd Stat/Ops5: seq(

```
seq
Expr:2N-4
Variable:N
start:1
end:10
step:
Paste
```

This will give the first 10 terms of the arithmetic sequence $a_n = 2n - 4$.

TI 83: 2nd Stat/Ops5:seq(
Seq(2n-4,n,1,10)

Finding Recursive terms on the Calculator both TI 83 and TI 84

Find the first 5 terms of the recursive sequence:
$a_1 = 32,\ \ a_n = a_{n-1}(1/2)$

Home Screen

Type 32 ENTER
2nd (-) * (1/2) ENTER
hit ENTER 3 more times

```
32
                32
Ans*(1/2)
                16
Ans*(1/2)
                 8
Ans*(1/2)
                 4
Ans*(1/2)
                 2
```

Formulas for Sequences and Series	
Arithmetic	Geometric
*Sequence: $a_n = a_1 + d(n-1)$	*Sequence: $a_n = a_1 r^{n-1}$
Series: $S_n = \frac{n}{2}\left(a_1 + a_n\right)$	*Series: $S_n = \frac{a_1 - a_1 r^n}{1 - r}$
*on regents exam reference sheet	*on regents exam reference sheet

Module 3 – Exponential and Logarithmic Functions

1. $1, 3, 5, 7, 9 \ldots$

Arithmetic or Geometric?	Explicit Formula	Recursive Formula
	$a_n =$	$a_1 =$ $a_n =$

Find Sum of Series with Formula	Find Sum of Series with Calculator

2. $2, 4, 8, 16, 32 \ldots$

Arithmetic or Geometric?	Explicit Formula	Recursive Formula
	$a_n =$	$a_1 =$ $a_n =$

Find Sum of Series with Formula	Find Sum of Series with Calculator

3. $-2,\ 1,\ 4,\ 7,\ 10\ldots$

Arithmetic or Geometric?	Explicit Formula	Recursive Formula
	$a_n =$	$a_1 =$ $a_n =$

Find Sum of Series with Formula	Find Sum of Series with Calculator

4. $1,\ 10,\ 100,\ 1000,\ 10000\ldots$

Arithmetic or Geometric?	Explicit Formula	Recursive Formula
	$a_n =$	$a_1 =$ $a_n =$

Find Sum of Series with Formula	Find Sum of Series with Calculator

Practice:

1. Given the sequence $23,\ 115,\ 575,\ 2875,\ \ldots$
   (a) Find the 6th term.
   (b) Find the sum of the first 8 terms.

2. Using sigma notation, express this exact sequence
   $-6,\ -4,\ -2,\ 0,\ 2,\ 4,\ 6$

3. Write a recursive formula to create this sequence
   $80,\ 240,\ 720,\ \ldots$

4. Use the formula $S_n = \dfrac{a_1 - a_1 r^n}{1 - r}$ to find the sum of the first 10 terms of the geometric sequence $3, 18, 108, \ldots$

Module 4 – Inferences and Conclusions from Data

Evaluating Simple Probability

1. Mary chooses an integer at random from 1 to 6. What is the probability that the integer she chooses is a prime number?

2. A box contains six black balls and four white balls. What is the probability of selecting a black ball at random from the box?

3. A six-sided number cube has faces with the numbers 1 through 6 marked on it. What is the probability that a number less than 3 will occur on one toss of the die?

4. Which event has a probability of zero?

   (1) choosing a triangle that is both isosceles and right
   (2) choosing a number that is greater than 6 and even
   (3) choosing a letter from the alphabet that has line symmetry
   (4) choosing a pair of parallel lines that have unequal slopes

5. Which inequality represents the probability, x, of any event happening?

   (1) $0 \leq x \leq 1$ (2) $0 < x < 1$ (3) $x \geq 0$ (4) $x < 1$

6. Throughout history, many people have contributed to the development of mathematics. These mathematicians include, Pythagoras, Euclid, Hypatia, Eucler, Einstein, Agnesi, Fibonacci, and Pascal. What is the probability that a mathematician's name selected at random from those listed with start with either the letter E or the letter A?

7. The faces of a cube are numbered from 1 to 6. What is the probability of *not* rolling a 5 on a single toss of this cube?

8. If the probability that it will rain on Thursday is $\frac{5}{6}$, what is the probability that it will *not* rain on Thursday?

9. A box contains 6 dimes, 8 nickels, 12 pennies, and 3 quarters. What is the probability that a coin drawn at random is *not* a dime?

10. John is playing cards. What is the probability of pulling an ace or queen from the deck?

11. A bag contains 19 chocolates; 5 are milk chocolate, 7 are kit kats, and 8 are bourville chocolate. If a chocolate is selected at random, what is the probability that the chocolate chosen is either milk chocolate or kit kat?

12. In a group of 100 people; 25 were regular apple eaters and 35 did not like apples. Find the probability that a person picked at random from this group is either a regular apple eater or a person who did not like apples.

13. In a group of 45 boys; 17 own laptops, 19 own desktops computers, and 7 own both. Find the probability that a boy picked from this group at random owns either a laptop or a desktop computer.

14. Holly is going to draw two cards from a standard deck without replacement. What is the probability that the first card is a king and the second card is an ace?

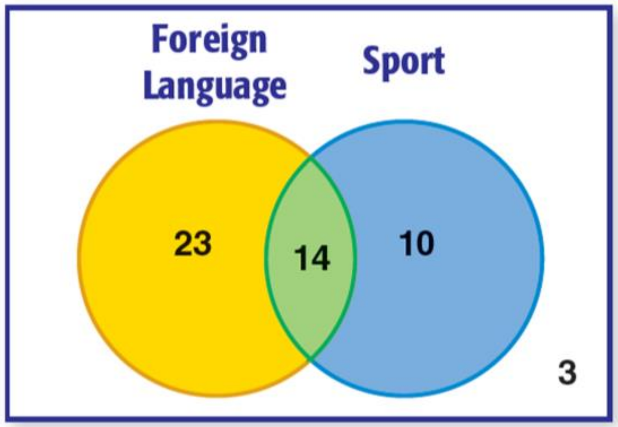
15. A jar contains colored stones that are 4 oink stones, 9 orange stones and 5 green stones. Ryan picks one stone, records its color and puts it back in the jar. Then he draws another stone. What is the probability of taking out an orange stone followed by a green stone?

16. Anna has 2 purple lipsticks, 3 red lipsticks and 3 pink lipsticks in her kit. She picks one lipstick, records its color, puts it back in the kit and draws another lipstick. What is the probability of taking out a purple lipstick followed by a red lipstick?

17. Caleb has 7 black caps, 4 yellow caps, and 9 blue caps in his wardrobe. Two caps are drawn without replacement from the wardrobe. What is the probability that both of the caps are blue?

18. Bella has 7 green beads, 10 yellow beads and 3 white beads in a pouch. Two beads are drawn without replacement from the pouch. What is the probability that both of the beads are yellow?

The data from a survey of students is shown in the Venn diagram. The students were asked whether or not they were taking a foreign language and whether or not they played a sport.

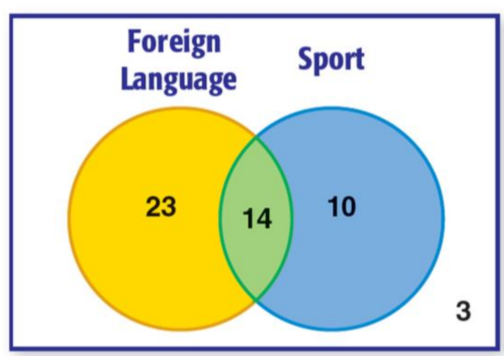


- How many students were surveyed?
- How many students play a sport? How many don't?
- How many students take a foreign language and play a sport?
- How many students don't take a foreign language and don't play a sport?

If a student is chosen at RANDOM from this survey, find the PROBABILITY of each of the following events.

In each of the following questions let F represent "student takes a foreign language" and let S represent "student plays a sport".

$P(S)$

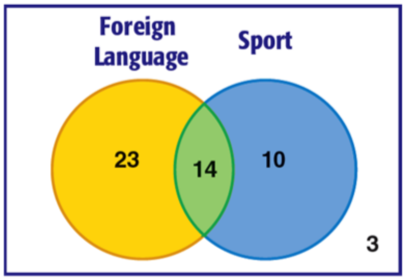


$P(S^c)$

$P(F \cap S)$

$P(F \cup S)$

Using the Venn diagram, complete the two way table.



	Play a Sport	Do Not Play a Sport	Total
Take a Foreign Language			
Do Not Take a Foreign Language			
Total			

Use the information in the table above to compute the following probabilities.

1. $P(F)$

2. $P(F^C)$

3. $P(S^C \cap F^C)$

4. $P(F \cup S)$

5. $P(F|S)$

6. $P(S|F)$

1. The Waldo School Board asked eligible voters to evaluate the town's library service. Data are summarized in the following table:

	How Would You Rate Our Town's Library Services?							
	Good		**Average**		**Poor**		**Do Not Use Library**	
Age (in years)	Male	Female	Male	Female	Male	Female	Male	Female
18–25	10	8	5	7	5	5	17	18
26–40	30	28	25	30	20	30	20	20
41–65	30	32	26	21	15	10	5	10
66 and Older	21	25	8	15	2	10	2	5

a. What is the probability that a randomly selected person who completed the survey rated the library as good?

b. Imagine talking to a randomly selected male voter who had completed the survey. How do you think this person rated the library services? Explain your answer.

c. Use the given data to construct a two-way table that summarizes the responses on gender and rating of the library services. Use the following template as your guide:

	Good	Average	Poor	Do Not Use	Total
Male					
Female					
Total					

d. Based on your table, answer the following:

i. A randomly selected person who completed the survey is male. What is the probability he rates the library services as good?

ii. A randomly selected person who completed the survey is female. What is the probability she rates the library services as good?

e. Based on your table, answer the following:

i. A randomly selected person who completed the survey rated the library services as good. What is the probability this person is male?

ii. A randomly selected person who completed the survey rated the library services as good. What is the probability this person is female?

2. Obedience School for Dogs is a small franchise that offers obedience classes for dogs. Some people think that larger dogs are easier to train and, therefore, should not be charged as much for the classes. To investigate this claim, dogs enrolled in the classes were classified as large (30 pounds or more) or small (under 30 pounds). The dogs were also classified by whether or not they passed the obedience class offered by the franchise. 45% of the dogs involved in the classes were large. 60% of the dogs passed the class. Records indicate that 40% of the dogs in the classes were small and passed the course.

   a. Complete the following hypothetical 1000 two-way table:

	Passed the Course	**Did Not Pass the Course**	**Total**
Large Dogs			
Small Dogs			
Total			

   b. Estimate the probability that a dog selected at random from those enrolled in the classes passed the course.

   c. A dog was randomly selected from the dogs that completed the class. If the selected dog was a large dog, what is the probability this dog passed the course?

   d. A dog was randomly selected from the dogs that completed the class. If the selected dog is a small dog, what is the probability this dog passed the course?

   e. Do you think dog size and whether or not a dog passes the course are related?

   f. Do you think large dogs should get a discount? Explain your answer.

Did male and female voters respond similarly to the survey question about building a new high school? Recall the original summary of the data.

	Should Our Town Build a New High School?					
	Yes		**No**		**No Answer**	
Age (in years)	Male	Female	Male	Female	Male	Female
18–25	29	32	8	6	0	0
26–40	53	60	40	44	2	4
41–65	30	36	44	35	2	2
66 and Older	7	26	24	29	2	0

1. Complete the following two-way frequency table:

	Yes	**No**	**No Answer**	**Total**
Male	119		6	
Female				
Total		230	12	515

2. Use the above two-way frequency table to answer the following questions:

   a. If a randomly selected eligible voter is female, what is the probability she will vote to build a new high school?

   b. If a randomly selected eligible voter is male, what is the probability he will vote to build a new high school?

3. An automobile company has two factories assembling its luxury cars. The company is interested in whether consumers rate cars produced at one factory more highly than cars produced at the other factory. Factory A assembles 60% of the cars. A recent survey indicated that 70% of the cars made by this company (both factories combined) were highly rated. This same survey indicated that 10% of all cars made by this company were both made at Factory B *and* were *not* highly rated.

a. Create a hypothetical 1000 two-way table based on the results of this survey by filling in the table below.

	Car Was Highly Rated by Consumers	Car Was Not Highly Rated by Consumers	Total
Factory A			
Factory B			
Total			

b. A randomly selected car was assembled in Factory B. What is the probability this car is highly rated?

Module 4 – Inferences and Conclusions from Data

Jack surveyed students at St. John's Prep. He found that 78 students own a cell phone and 57 of those students own an iPod Touch. There are 13 students that do not own a cell phone but own an iPod Touch. Nine students do not own either device. Complete the two-way table shown below:

	iPod Touch	No iPod Touch	Total
Cell Phone	57		78
No Cell Phone	13	9	
Total			

1. Find the probability that s student has a cell phone or an iPod Touch.

2. Find the probability that a student has a cell phone given that the student has an iPod Touch.

3. Find the probability that a student has an iPod Touch given that the student has a cell phone.

Health officials in Milwaukee, Wisconsin, were concerned about teenagers with asthma. People with asthma often have difficulty with normal breathing. In a local research study, researchers collected data on the incidence of asthma among students enrolled in a Milwaukee public high school.

	No Household Member Smokes	**At Least One Household Member Smokes**	**Total**
Student Has Asthma	73	120	193
Student Does Not Have Asthma	506	301	807
Total	579	421	1,000

1. Based on your completed two-way table, find the following probabilities as a decimal (rounded to three decimal places):

    a. A randomly selected student has at least one household member who smokes. What is the probability that this student has asthma?

    b. A randomly selected student does not have at least one household member who smokes. What is the probability that this student has asthma?

2. Do you think that whether or not a student has a household member who smokes is related to whether or not this student has asthma? Explain your answer.

Photos and videos have become an important part of the online social media experience. A survey shows that 52% of Internet users post photos that they have taken themselves, 26% of Internet users post videos that they have taken themselves and 24% of Internet users post both photos and videos that they have taken themselves. Let *A* represent users who post photos and *B* represent users who post videos.

a. What are the values of $P(A)$, $P(B)$, and $P(A \cap B)$

b. Draw a Venn diagram, and label it with the probabilities from part (a).

c. Use the conditional probability formula to calculate $P(A|B)$. (Round your answer to the nearest thousandth.)

d. Use the conditional probability formula to calculate $P(B|A)$. (Round your answer to the nearest thousandth.)

e. Which is greater, $P(A|B)$ or $P(A)$? Explain why this is relevant.

Two events are INDEPENDENT when knowing that one event occurred DOES NOT change the likelihood that the second event has occurred.

$$P(A|B) = P(A)$$

	Participate in After School Athletics (A)	Does Not Participate in After School Athletics (A^c)	Total
Females (F)	232	348	580
Males (M)	168	252	420
Total	400	600	1000

Find the probability that a randomly selected student participates in athletics after school.

Find the probability that a randomly selected student who is female participates in athletics after school.

Find the probability that a randomly selected student who is male participates in athletics after school.

Module 4 – Inferences and Conclusions from Data

1. The following hypothetical 1000 two-way table was introduced in the previous lesson:

	Plan to Vacation in New York Within the Next Year	Do Not Plan to Vacation in New York Within the Next Year	Total
Watched the Online Ad	300	450	750
Did Not Watch the Online Ad	100	150	250
Total	400	600	1,000

Are the events "a randomly selected person watched the online ad" and "a randomly selected person plans to vacation in New York within the next year" independent or not independent? Justify your answer using probabilities calculated from information in the table.

2. A survey conducted at a local high school indicated that 30% of students have a job during the school year. If having a job and being in the eleventh grade are not independent, what do you know about the probability that a randomly selected student who is in the eleventh grade would have a job? Justify your answer.

3. Eighty percent of the dogs at a local kennel are in good health. If the events "a randomly selected dog at this kennel is in good health" and "a randomly selected dog at this kennel weighs more than 30 pounds" are independent, what do you know about the probability that a randomly selected dog that weighs more than 30 pounds will be in good health? Justify your answer.

Module 4 – Inferences and Conclusions from Data

1. Of the light bulbs available at a store, 42% are fluorescent, 23% are labeled as long life, and 12% are fluorescent *and* long life.

   a. A light bulb will be selected at random from the light bulbs at this store. Rounding your answer to the nearest thousandth where necessary, find the probability that

      i. The selected light bulb is not fluorescent.

      ii. The selected light bulb is fluorescent given that it is labeled as long life.

   b. Are the events "fluorescent" and "long life" independent? Explain.

2. When a person is selected at random from a very large population, the probability that the selected person is right-handed is 0.82. If three people are selected at random, what is the probability that

   c. They are all right-handed?

   d. None of them is right-handed?

Module 4 – Inferences and Conclusions from Data

1. Of the works of art at a large gallery, 59% are paintings, and 83% are for sale. When a work of art is selected at random, let the event that it is a painting be A and the event that it is for sale be B.

   a. What are the values of $P(A)$ and $P(B)$?

   b. Suppose you are told that $P(A \text{ and } B) = 0.51$. Find $P(A \text{ or } B)$.

2. A traveler estimates that, for an upcoming trip, the probability of catching malaria is 0.18, the probability of catching typhoid is 0.13, and the probability of catching neither of the two diseases is 0.75.

   a. Draw a Venn diagram to represent this information.

   b. Calculate the probability of catching both of the diseases.

   c. Are the events "catches malaria" and "catches typhoid" independent? Explain your answer.

3. A deck of 40 cards consists of the following:

   - 10 black cards showing squares, numbered 1–10
   - 10 black cards showing circles, numbered 1–10
   - 10 red cards showing X's, numbered 1–10
   - 10 red cards showing diamonds, numbered 1–10

   A card will be selected at random from the deck.

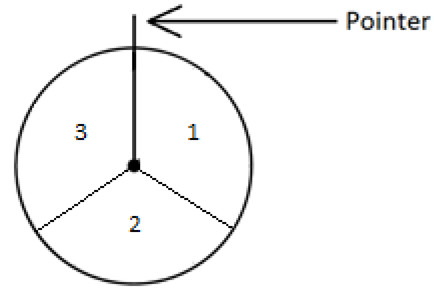
   a. i. Are the events "the card shows a square" and "the card is red" disjoint? Explain.

      ii. Calculate the probability that the card will show a square or will be red.

   b. i. Are the events "the card shows a 5" and "the card is red" disjoint? Explain.

      ii. Calculate the probability that the card will show a 5 or will be red.

4. The diagram below shows a spinner. When the pointer is spun, it is equally likely to stop on 1, 2, or 3. The pointer will be spun three times. Expressing your answers as fractions in lowest terms, find the probability, and explain how the answer was determined that the total of the values from all thee spins is



a. 9.

b. 8.

c. 7.

5. A number cube has faces numbered 1 through 6, and a coin has two sides, heads and tails. The number cube will be rolled once, and the coin will be flipped once. Find the probabilities of the following events. (Express your answers as fractions in lowest terms.)

a. The number cube shows a 6.

b. The coin shows heads.

c. The number cube shows a 6, and the coin shows heads.

d. The number cube shows a 6, or the coin shows heads.

6. Kevin will soon be taking exams in math, physics, and French. He estimates the probabilities of his passing these exams to be as follows:

- Math: 0.9
- Physics: 0.8
- French: 0.7

Kevin is willing to assume that the results of the three exams are independent of each other. Find the probability of each event.

a. Kevin will pass all three exams.

b. Kevin will pass math but fail the other two exams.

c. Kevin will pass exactly one of the three exams.

Module 4 – Inferences and Conclusions from Data

Using Statistics

DATA collected from a SAMPLE is analyzed numerically and graphically to make INFERENCES about a POPULATION.

	POPULATION	SAMPLE
	SUMMARY measures are called CHARACTERISTICS or PARAMETERS.	SUMMARY measures are called STATISTICS.
MEAN	μ	$\bar{x}$
STANDARD DEVIATION	σ	s_x

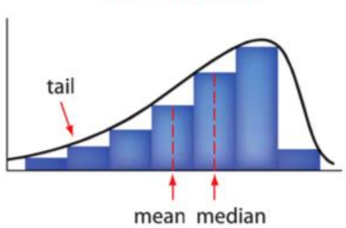
Congress wants to know whether ALL adults in the US 18 years and older are in favor of gun control.

What is the population of interest?

Is it possible to poll the entire population?

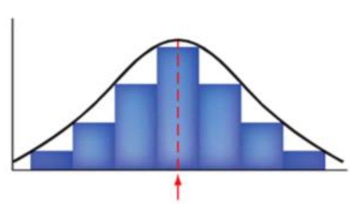
KeyConcept Symmetric and Skewed Distributions

SKEWED LEFT DISTRIBUTION



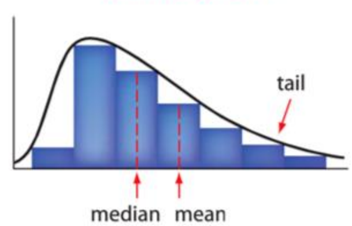
- The mean is less than the median.
- The majority of the data are on the right of the mean.

SYMMETRIC DISTRIBUTION



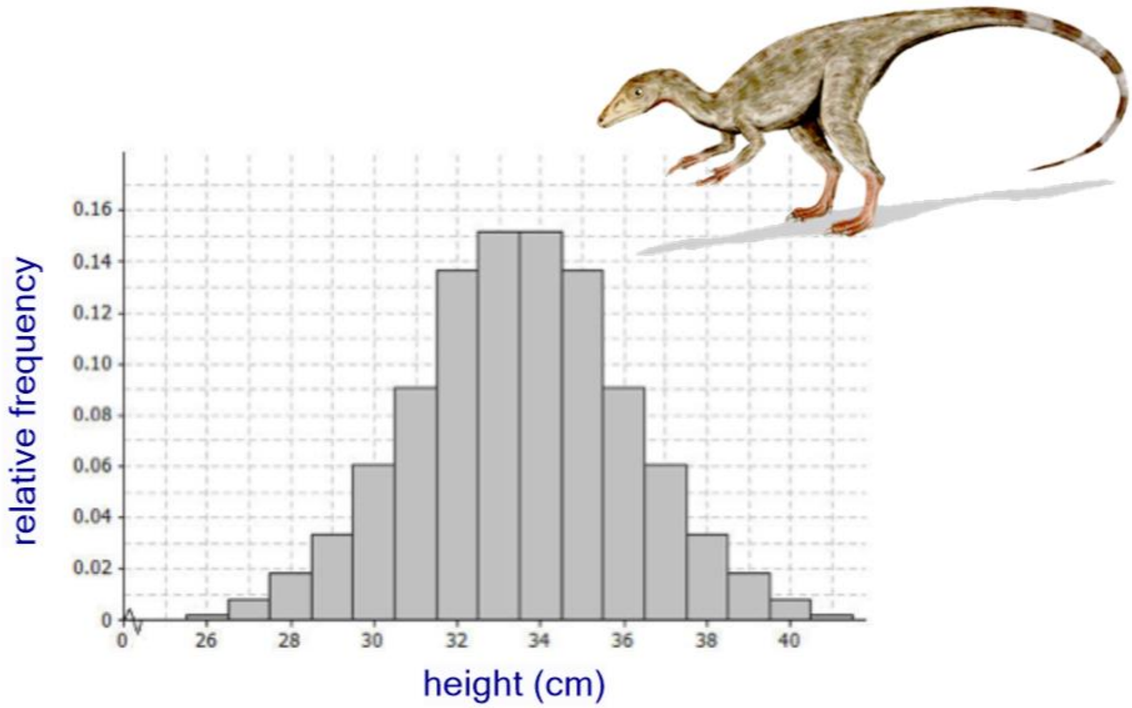
- The mean and median are approximately equal.
- The data are evenly distributed on both sides of the mean.

SKEWED RIGHT DISTRIBUTION

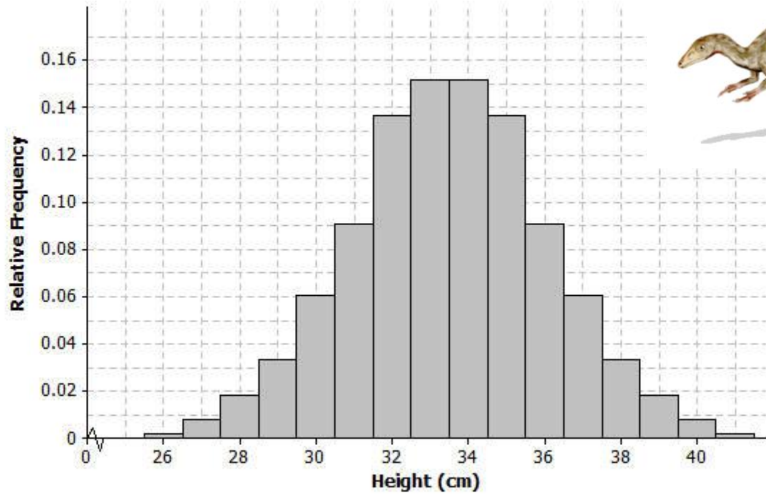


- The mean is greater than the median.
- The majority of the data are on the left of the mean.

A paleontologist studies prehistoric life and sometimes works with dinosaur fossils. The histogram below shows the distribution of heights (rounded to the nearest centimeter) of 660 "compys".



The following is a relative frequency histogram of the compy heights:



7. What does the relative frequency of 0.136 mean for the height of 32 cm?

8. What is the width of each bar? What does the height of the bar represent?

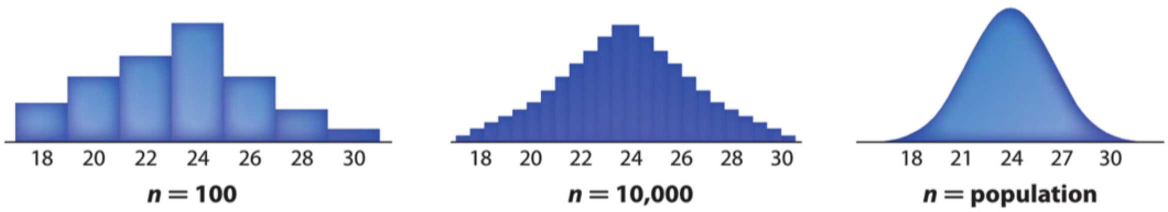
9. What is the area of the bar that represents the relative frequency for compys with a height of 32 cm?

10. If a compy were selected randomly from these 660 compys what is the probability that its height would be 32 cm?

11. What is the sum of all the areas of the bars in the histogram? Why?

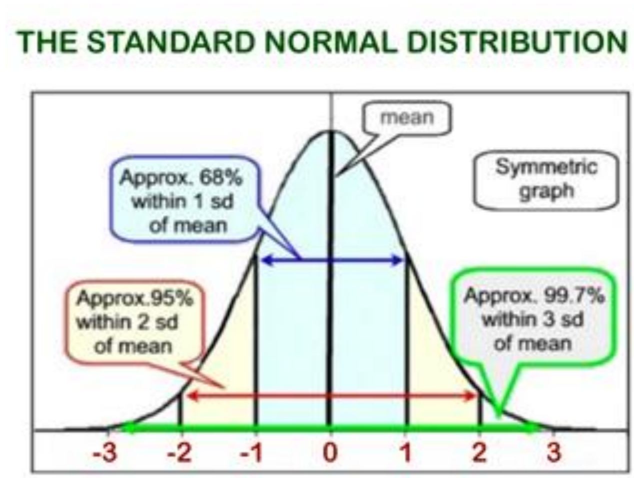
12. The mean compy height is 33.5 cm and the standard deviation is 2.56 cm. Interpret the mean and the standard deviation in the context of this problem.

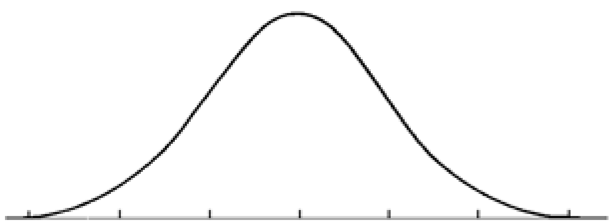
As SAMPLE sizes increase distributions become more SYMMETRIC and can be modeled by a smooth curve.

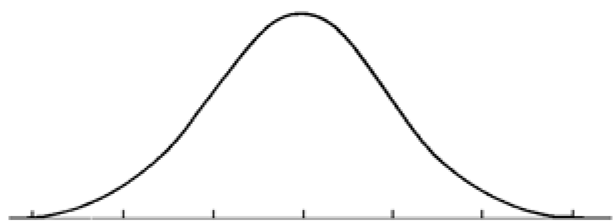


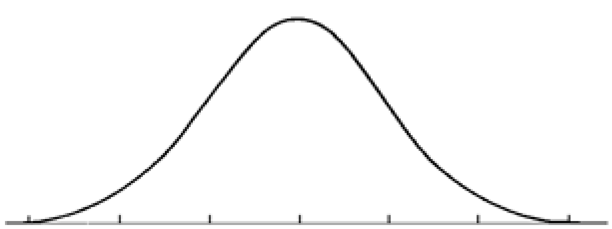
<u>Properties of the Normal Distribution (Bell Curve)</u>

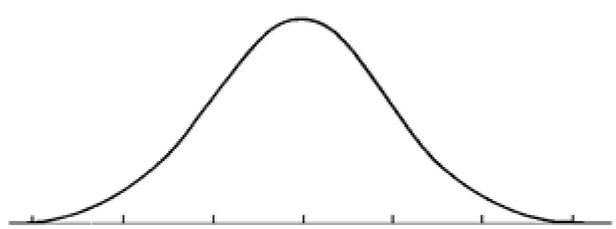
- The graph of the curve is bell shaped and SYMMETRIC with respect to the MEAN.
- The MEAN and MEDIAN are equal and located at the center.
- The TOTAL AREA under the curve is equal to 1.
- The STANDARD DEVIATION is the appropriate measure for variability.





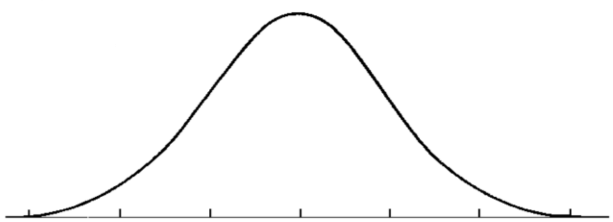


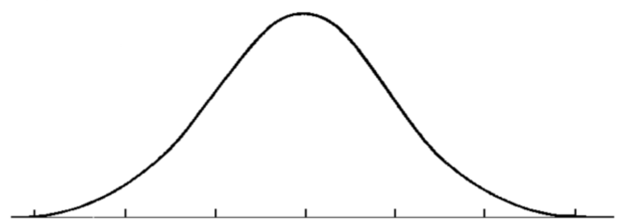




- The total area under the curve is equal to 1. This is the total probability of the entire distribution.
- Almost all of the area is within 3 standard deviations of the mean.
- The distribution is symmetric.
- The mean is 0 and the standard deviation is 1.
- The number of standard deviations from the mean is called a z-score.
- The standard normal distribution shows $\mu = 0$ and z-scores.

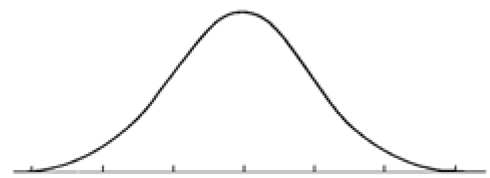
SAT and ACT scores are normally distributed. Lauren took both tests and is deciding which score to send to her preferred college. She scored 1360 on the SAT and 30 on the ACT. The SAT had a mean score of 1000 with a standard deviation of 180 and the ACT had a mean score of 20 with a standard deviation of 5.



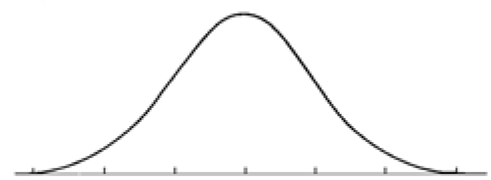


Module 4 – Inferences and Conclusions from Data

1. If Lauren scored 20 on the ACT what would be her percentile?

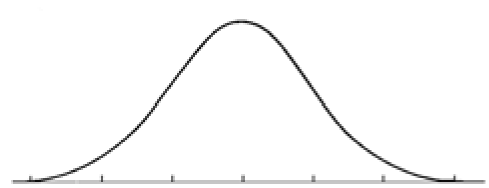


2. If Lauren's z-score were a -1, what did she score on the ACT and what was her percentile?



3. What percent of student scored greater than 30 on this exam?

4. What percent of students scored between 10 and 25 on this exam?

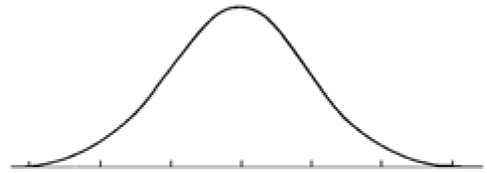


Module 4 – Inferences and Conclusions from Data

Using Z-Scores

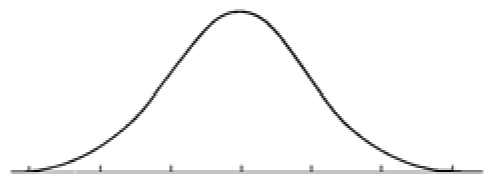
A survey taken to determine the average number of friends of 820 FACEBOOK users was normally distributed with a mean of 38 and a standard deviation of 12.

If a user is chosen at random what is the probability that this user has MORE than 44 friends?



A survey taken to determine the average number of friends of 820 FACEBOOK users was normally distributed with a mean of 38 and a standard deviation of 12.

If a user is chosen at random what is the probability that this user has FEWER than 52 friends?

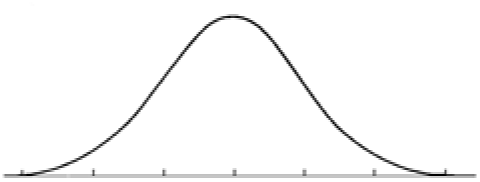


$$z = \frac{\text{value} - \text{mean}}{\text{standard deviation}}$$

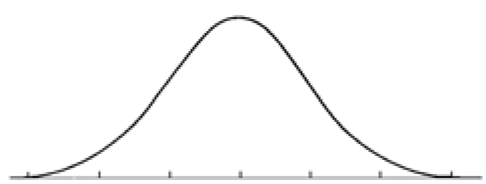
Practice

FACEBOOK with CALCULATOR: MEAN of 38 and STANDARD DEVIATION of 12. You MUST draw the curve.

1. $P(x > 48) =$



2. $P(20 \leq x \leq 30) =$



Module 4 – Inferences and Conclusions from Data

When answering the questions below include a drawing of a labeled normal curve (scores and z-scores) and shade the appropriate area for each problem.

1. A survey asked a group of high school students about the number of hours of sleep they receive in a 24 hour period. The data was normally distributed with a mean of 7 hours and a standard deviation of 1.25 hours.

   (a) If George slept 8.25 hours, what percent of the students in the survey slept less than George? What percent slept more than George?

   (b) If your math teacher slept 5.75 hours, what percent of students in the survey slept more than the teacher?

   (c) What percent of students slept between 5.75 and 8.25 hours?

   (d) What interval about the mean includes 95% of the data in the survey?

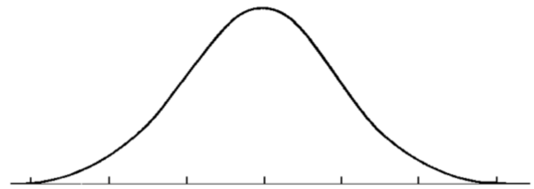
2. The time spent by groups of employees working on new Apple projects has a mean of 18 hours and a standard deviation of 4 hours.

   (a) What is the probability that the amount of time spent by a group is less than 10 hours?

   (b) What is the probability that the amount of time spent by a group is more than 26 hours?

   (c) What is the probability that the amount of time spent by a group is between 14 and 22 hours?

1. The lifespan of a toaster is normally distributed with a mean of 14 years and a standard deviation of 2.1 years.



(a) Label the normal curve with the mean, and standard deviations of ± 1, ± 2, and ± 3 .

(b) Shade the region that represents the proportion of toasters that last longer than 17 years.

(c) Find the z-score for 17 years.

(d) Find the probability that a toaster lasts longer than 17 years.

2. The number of texts sent per day by a sample of 811 teens is normally distributed with a mean of 48 and a standard deviation of 16.

(a) Calculate the z-score for 40 texts.

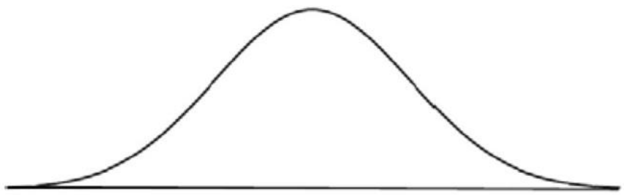
(b) Find the probability that a teen chosen at random sends fewer than 40 texts. (Draw and shade the curve)

(c) How many teens from this sample send fewer than 40 texts?

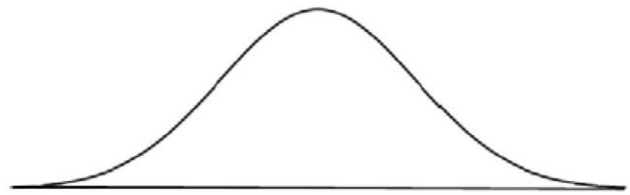
(d) How many teens in this sample sent between 45 and 75 texts? (Draw and shade the curve)

3. For the following questions, on the given curve, mark the mean and mark the given z-scores. Shade the area needed. Find the probability of the shaded area.

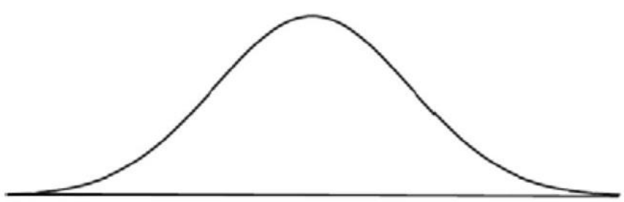
(a) The area to the left of $z = -1.45$.



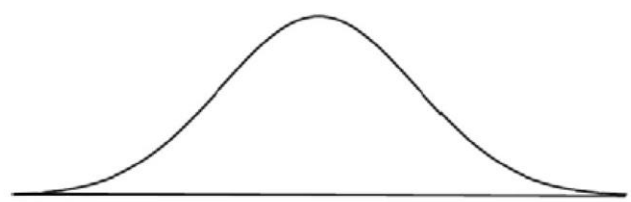
(b) The area to the right of $z = 2.08$.



(c) The area between $z = -2.37$ and $z = -.88$



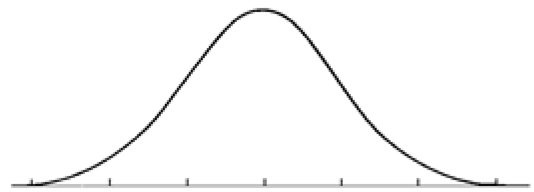
(d) The area between $z = .72$ and $z = 1.24$



Module 4 – Inferences and Conclusions from Data

1. The distribution of heights of adult American men is approximately normal with a MEAN of 69 inches and a STANDARD DEVIATION of 2.5 inches.

   What is the minimum height of a man if he is in the tallest 10% of all men?

   Shade in the appropriate AREA on the curve.

   Suppose you were given the following choices:

   (1) 69 (2) 65 (3) 73 (4) 78

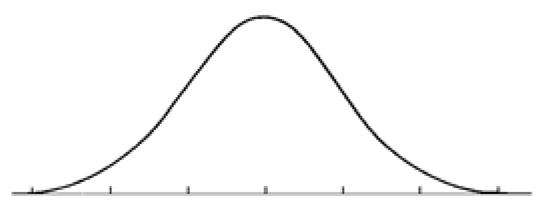
   Explain your reasoning.

   Find the exact value with the calculator.

2. Almost all high school students who intend to go to college take the SAT. In a recent test, the MEAN SAT score (verbal and math) of all students was 1020 with a STANDARD DEVIATION of 153. What is the minimum score a student can get on the SAT to be in the top 10% of the exam?

   (1) 1173 (2) 1216 (3) 1326 (4) 1479

   Be ready to defend your choice!

1. SAT scores were originally scaled so that the scores for each section were approximately normally distributed with a mean of 500 and a standard deviation of 100. Assuming that this scaling still applies, use a table of standard normal curve areas to find the probability that a randomly selected SAT student scores

   a. More than 700.

   b. Between 440 and 560.

2. In 2012, the mean SAT math score was 514, and the standard deviation was 117. For the purposes of this question, assume that the scores were normally distributed. Using a graphing calculator, and without using z-scores, find the probability (rounded to the nearest thousandth), and explain how the answer was determined that a randomly selected SAT math student in 2012 scored

   a. Between 400 and 480.

   b. Less than 350.

Module 4 – Inferences and Conclusions from Data

Summary of Types of Statistical Studies

- **There are three major types of statistical studies: observational studies, surveys, and experiments.**
  - **An *observational study* records the values of variables for members of a sample.**
  - **A *survey* is a type of observational study that gathers data by asking people a number of questions.**
  - **An *experiment* assigns subjects to treatments for the purpose of seeing what effect the treatments have on some response.**
- **To avoid bias in observational studies and surveys, it is important to select subjects randomly.**
- **Cause-and-effect conclusions cannot be made in observational studies or surveys.**
- **In an experiment, it is important to assign subjects to treatments randomly in order to make cause-and-effect conclusions.**

Use the following three situations for problems 1, 2, and 3.

A. Researchers want to determine if there is a relationship between whether or not a woman smoked during pregnancy and the birth weight of her baby. Researchers examined records for the past five years at a large hospital.

B. A large high school wants to know the proportion of students who currently use illegal drugs. Uniformed police officers asked a random sample of 200 students about their drug use.

C. A company develops a new dog food. The company wants to know if dogs would prefer its new food over the competition's dog food. One hundred dogs, who were food deprived overnight, were given equal amounts of the two dog foods: the new food vs. competitor's food. The proportion of dogs preferring the new food was recorded.

1. Which situation above describes an experiment? Explain why.

2. Which situation describes a survey? Will the result of the survey be accurate? Why or why not?

3. The remaining situation is an observational study. Is it possible to perform an experiment to determine if a relationship exists? Why or why not?

Module 4 – Inferences and Conclusions from Data

1. A Gallup Poll in November found that 51% of people in the sample wanted to lose weight. Gallup announced that "the margin of sampling error is $\pm 4\%$."

   Find the confidence interval for this poll.

2. A sampling distribution for the BMI of women aged 20-29, reported an average body mass index of 26.8 with a standard deviation of 0.3.

   a) Find a 95% confidence interval by finding the values $\left(\bar{x} - 2s_x\ ,\ \bar{x} + 2s_x\right)$.

   b) What is the margin of error in this confidence interval?

3. A 95% confidence interval for the mean number of slices of pizza able to be eaten at one meal by 17 year old males was reported as $\left(3.5\ ,\ 4.5\right)$.

   a) Based on this confidence interval, what was the value of the estimate of the mean, $\bar{x}$?

   b) What is the margin of error?

   c) If the sample size were doubled, what would happen to the mean?

   d) If the sample size were doubled, what would happen to the standard deviation?

Notes

Notes